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TRIANGULATION—A PRECISE METHOD FOR SATELLITE TRACKING

BY

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INTRODUCTION

The advent of artificial satellites introduced a difficult astronomical problem, that of the precise determination of the orbits of artificial satellites.

Although this problem has been called new, it is an old problem made more difficult by the gross exaggeration of natural phenomena to the extent that they are often not immediately recognizable as such. *Parallax*, which must be considered in the observation of the natural moon and the planets of the solar system, becomes so great for satellites close to the Earth that two observers 100 miles apart may see the satellite in entirely different regions of the sky. The *eclipsing* of the natural moon is a rare occurrence compared with that of an artificial satellite close to the Earth, which is eclipsed about half its life time when it travels behind the Earth's shadow. Natural celestial objects are *occulted* by the Earth approximately half the time, whereas an artificial satellite close to the Earth is occulted when it travels below the observer's horizon, which is most of the time. The *apparent angular* velocity of artificial satellites is much greater than that of natural astronomical bodies, greatly emphasizing the need for speed in making measurements. Large telescopes cannot be "whipped around" to follow artificial satellites. The *phase* of an artificial satellite can change from new moon to crescent to gibbous to full moon in less than 5 minutes, or remain essentially the same throughout its entire visible path. Fluctuations in apparent brightness due to tumbling are most noticeable near the crescent phase.

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The general problem in measurement of a satellite orbit is to obtain the true position and motion of the satellite from measurements of its apparent position and motion.

Present techniques of observation employ either radio or optical means. The latter is capable of a much higher degree of precision with equipment of manageable size, but is limited by the fact that the opportunity for visual observation is much less than that for radio observation.

Data reduction techniques can be grouped into four general categories: (1) triangulation, (2) radar (radio direction and range), (3) stadia, and (4) Doppler-shift methods. Optical triangulation is capable of the highest degree of precision.

Triangulation and radar methods are "logically pure" in that no laws of kinematics (involving non-central forces of a non-homogeneous Earth and drag forces of a largely unknown atmosphere) need be assumed in the reduction of the data.

The "stadia" and Doppler-shift methods are often simpler methods of data reduction. The simplest form of the "stadia" method consists of measuring the greatest apparent angular velocity of a satellite for a given passage, which occurs at the point of nearest approach. This is related to the *approximate velocity* of the satellite found from period measurements through the use of Kepler's Law which holds for systems of point-masses or spheres whose mass distribution is a function of the radius only. If the "stadia" measurement is not obtained near the highest point of the apparent path, the calculations become decidedly more complex and the advantage of simplicity is lost altogether. The Doppler-shift method (1)[2] relates measured Doppler-shift records to the radial component of velocity of a satellite. True position and motion of the satellite is found by the use of Kepler's Law or by making educated judgments. The Doppler-shift method, as it has been used, requires an active satellite-borne transmitter. The use of reflected radio waves for Doppler measurements appears possible in the future.

Of these methods, only the triangulation and radar methods can be used to determine the paths of non-ballistic objects or ballistic objects whose "initial conditions" are not known.

The foundations of any measurement rest on theories. Stadia measurement makes use of the Newtonian theory with assumptions concerning the shape and mass distribution of the Earth. Radar methods require assumptions concerning propagation of electromagnetic radiation (2). The method of photographic triangulation makes use of axioms of geometry and optics, and requires only a moderate assumption concerning the shape of the Earth, and, for stations not too widely separated, very little of that.

Data not obtained simultaneously usually lead to what is essentially

[2] The boldface numbers in parentheses refer to the references appended to this paper.

a stadia reduction in the initial stages of orbit determination, and, subsequently, to corrections to approximate orbits.

Although triangulation has been used in surveying for centuries, its application to determining the orbits of satellites is new and worthy of review. The triangulation is essentially in a spherical coordinate system, as shown in Fig. 1.

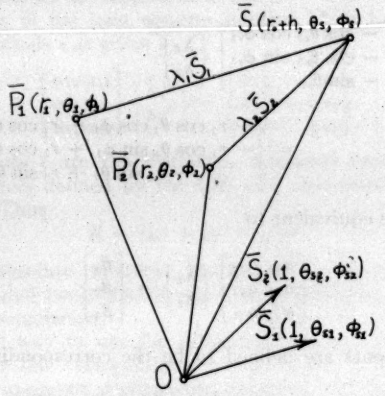

FIG. 1. Vector triangulation.

THE THEORY OF TRIANGULATION

Let P_1 and P_2 be the positions of two triangulation stations, O be the center of the Earth, S be the instantaneous satellite position, and $\mathbf{S}_1$ and $\mathbf{S}_2$ be unit direction vectors in the direction of the apparent positions of the satellite as measured by the two stations, as shown in the figure.

Subsequently, these points will be treated as position vectors as well as points.

If a geocentric spherical coordinate system as well as a Cartesian coordinate system are set up so that the z-axis passes through the North Pole, with the x–y plane being the equatorial plane, any vector $\mathbf{V}$ can be expressed in the form

$$\mathbf{V} = iV_x + jV_y + kV_z$$

where i, j, k are unit vectors in the Cartesian system,

$$\begin{aligned} V_x &= |\mathbf{V}| \cos\theta \cos\phi \\ V_y &= |\mathbf{V}| \cos\theta \sin\phi \\ V_z &= |\mathbf{V}| \sin\theta, \end{aligned}$$

and $|\mathbf{V}|$ is the magnitude of $\mathbf{V}$.

Referring to Fig. 1,

$$\mathbf{S} = \mathbf{P}_1 + \lambda_1 \mathbf{S}_1 = \mathbf{P}_2 + \lambda_2 \mathbf{S}_2 \tag{1}$$

where λ_1 and λ_2 are scalar constants to be determined. Equating the vector components of Eq. 1 yields the matrix equation

$$\begin{bmatrix} \cos\theta_{s1}\cos\phi_{s1} & -\cos\theta_{s2}\cos\phi_{s2} \\ \cos\theta_{s1}\sin\phi_{s1} & -\cos\theta_{s2}\sin\phi_{s2} \\ \sin\theta_{s1} & -\sin\theta_{s2} \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \end{Bmatrix}$$

$$= \begin{Bmatrix} -r_1\cos\theta_1\cos\phi_1 + r_2\cos\theta_2\cos\phi_2 \\ -r_1\cos\theta_1\sin\phi_1 + r_2\cos\theta_2\sin\phi_2 \\ -r_1\sin\theta_1 + r_2\sin\theta_2 \end{Bmatrix}. \tag{2}$$

This equation is equivalent to

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \end{Bmatrix} = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{Bmatrix} \tag{3}$$

where the elements are defined to be the corresponding elements of Eq. 2.

It is immediately apparent that this system is over-determined. For almost all satellite and station positions any two rows are independent and can be solved simultaneously for the λ's, from which $\mathbf{S}$ can be determined by means of Eq. 1.

This redundancy of data can be used to advantage in either of two ways. The first is to make a least square error solution of the over-determined system. (2) has the form

$$(A)\{\lambda\} = \{B\},$$

where (A) is the $(3, 2)$ matrix, $\{\lambda\}$ the $(2, 1)$ matrix, and $\{B\}$ the $(3, 1)$ matrix. If $(A)^T$ is the transposed matrix of (A), the solution of

$$(A)^T(A)\{\lambda\} = (A)^T\{B\} \tag{4}$$

can be shown to yield the least square error solution for the λ's (3). The least square solution for the λ's is

$$\lambda_1^* = \frac{\delta_1 - \delta_2\gamma}{1 - \gamma^2}$$

$$\lambda_2^* = \frac{\delta_2 - \delta_1\gamma}{1 - \gamma^2} \tag{5}$$

where

$$\gamma = \alpha_{11}\alpha_{12} + \alpha_{21}\alpha_{22} + \alpha_{31}\alpha_{32},$$
$$\delta_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 + \alpha_{31}\beta_3,$$

and

$$\delta_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 + \alpha_{32}\beta_3,$$

where the α's and β's are defined as in Eq. 3.

Substitution of the least squares solution λ^* of Eq. 5 into Eq. 3 defines the residuals ϵ as given by

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{Bmatrix} \lambda_1^* \\ \lambda_2^* \end{Bmatrix} - \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{Bmatrix} = \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{Bmatrix}. \tag{6}$$

The residuals ϵ are the distances, measured componentwise, by which the vectors defined by the left- and right-hand sides of Eq. 1 fail to meet. Thus

$$E = (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)^{1/2}, \tag{7}$$

which is the absolute difference between the position vectors of the satellite computed two different ways, is an indication of the absolute error of the triangulation.

The second way to use the redundancy is to transfer timing information from one triangulation station to the other by pairing of instantaneous apparent positions on apparent loci observed from the two stations. This makes a master-station–slave-station organization possible, with obvious economies in apparatus. Only the master station needs to record timing information concerning the apparent path of the satellite as observed from its position.

A graphical scheme for performing this point-pairing has been developed. Consider Fig. 2, which represents a unit sphere on which the two positions of the observers are located by points P_1 and P_2. Let the instantaneous position of the satellite be S and the sub-satellite point (central projection of S on the unit sphere) be S'. Then, the apparent angular positions determined by the two stations are the directions established by the lines P_1S and P_2S. The triangulation problem can now be stated: Knowing the observer's positions, P_1 and P_2, and the apparent positions of the satellite S_1 and S_2 as measured simultaneously by the two observers, find the coordinates of S' and the height of the satellite above the Earth (length of the line $S'S$).

Construct radius vectors from the center of the sphere in the directions of the apparent positions of the satellite as seen by the observers. Thus, in Fig. 2,

$$OS_1 \,||\, P_1S$$
$$OS_2 \,||\, P_2S,$$

and S_1 and S_2 are the projections of these vectors on the unit sphere.

It is evident that points OP_1SS_1 and OP_2SS_2 are co-planar, respectively, and that OS is the intersection of the two planes, and that further, S', the sub-satellite point, is the intersection of OS and the unit sphere.

Now, the intersections of the planes OP_1SS_1 and OP_2SS_2 with the unit sphere are great circles since they contain the center of the sphere. Consequently, S', the sub-satellite point, can be found as the intersection of great circles which contain P_1S_1 and P_2S_2.

As the satellite S moves, the points S_1 and S_2 move on the unit sphere in paths which are related to each other pointwise, as shown by the following construction.

Let Q be the intersection of great circles containing the pairs of points P_1P_2 and S_1S_2.

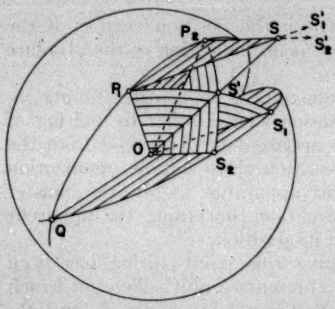

FIG. 2. Triangulation from a unit sphere.

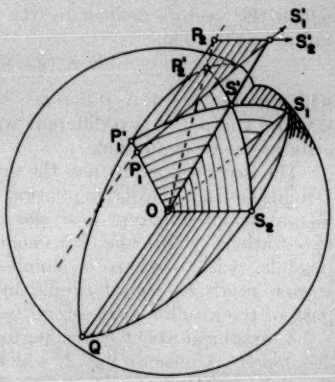

FIG. 3. Triangulation from stations not on a unit sphere.

Then it follows that plane P_1P_2S and plane OS_1S_2Q are parallel planes, since, by construction, $OS_1 \| P_1S$ and $OS_2 \| P_2S$.

Straight lines P_1P_2 and OQ are intersections of these two parallel planes with plane OQP_1P_2. Therefore, $OQ \| P_1P_2$.

Since P_1 and P_2 are fixed station locations, it follows that Q is fixed and can be found from the following equation by attaching vector significance to all points on the unit sphere:

$$\mathbf{Q} = \frac{\mathbf{P}_1 - \mathbf{P}_2}{|\mathbf{P}_1 - \mathbf{P}_2|}. \tag{8}$$

Q has the significance of being the point through which all great circles connecting the instantaneous apparent positions of the satellite

must pass. This point is of importance because it allows pairing of apparent positions, and hence transfer of timing information from one triangulation station to the other.

The essential features of triangulation from a unit sphere are retained in case the triangulation stations are not equidistant from the center of the Earth, as can be seen from Fig. 3, and the appropriate modifications of the proof above. Q can still be found from Eq. 4, and the unit sphere merely serves as a surface on which the geometry of triangulation can be projected.

A more convenient form for this projection is the Gnomonic projection, which is a central projection of points on a sphere to a tangent plane. This mapping process transforms all great circles on the sphere to straight lines, as shown in Fig. 4.

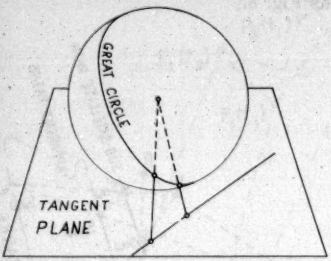

FIG. 4. The Gnomonic projection.

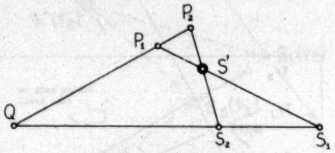

FIG. 5. A Gnomonic projection of Figs. 2 and 3.

The great circles $P_1'S_1$, $P_2'S_2$, $P_2'P_1'Q$, and S_1S_2Q, of Fig. 3, which serve to locate the sub-satellite point S', are mapped by the Gnomonic projection into a figure similar to Fig. 5.

This representation can be used both as a basis for theoretical discussions of triangulation, and also as a rapid, approximate method for reduction of triangulation data.

APPLICATIONS OF THE THEORY

As an example of the theoretical application of Fig. 5, consider the transfer of timing information from one observer's station to another.

Although triangulation data can be obtained by a variety of methods, photographic triangulation is graphic, and provides an excellent example for consideration. Typical pairs of triangulation photographs, if taken by fixed cameras and ideally superimposed, would have the appearance of Fig. 6. This figure, when replotted on a Gnomonic projection after the apparent positions are interpolated in the field of stars, appears as in Fig. 7, on which P_1, P_2 and Q of Fig. 3 are also plotted.

Either geocentric or celestial coordinates may be used, for, if the time is known, conversion between the two systems is simple. The points on the apparent paths (Tr 1 and Tr 2, observed from the stations P_1 and P_2) which correspond to the same instantaneous position of the satellite are $S_1(t_1)$ and $S_2(t_1)$; $S_1(t_2)$ and $S_2(t_2)$; and these correspond to the sub-satellite positions $S'(t_1)$ and $S'(t_2)$, respectively. Since there was no identifiable timing information on the satellite trace 2 (Tr 2), $S_2(t_1)$ and $S_2(t_2)$ have been located by transferring timing information from Station 1 to Station 2.

As a second theoretical application of the Gnomonic projection, consider a comparison of triangulation pairs obtained by right-ascension (or other) tracking cameras and cameras fixed to the Earth.

The projection for right-ascension drive cameras which corresponds to Fig. 7 for fixed cameras appears as Fig. 8.

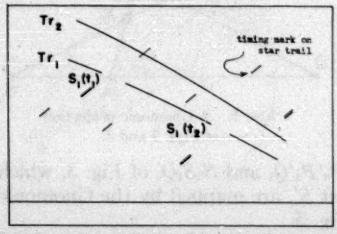

FIG. 6. Typical photographic triangulation data ideally superimposed.

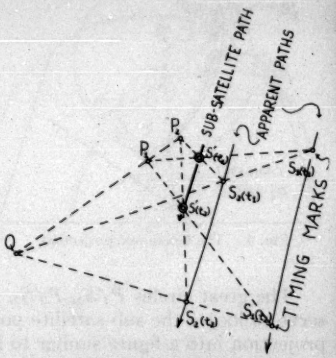

FIG. 7. Gnomonic projection of fixed camera data.

Point pairing in this case is more difficult than for data obtained by using fixed cameras because the pole Q and the observers' positions P_1 and P_2 are functions of time. Both satellite traces must contain all the necessary timing information since the star images cannot carry this information (in contrast to fixed camera photographs). If these time functions are not known, or are ambiguious, as often happens when timing markers are lost because of fluctuations in brightness caused by tumbling of non-spherical satellites, point pairing and triangulation may not be possible.

In contrast, cameras fixed to the Earth yield pairs of traces which can always be matched pointwise to secure triangulations from which the path of the satellite over the Earth can be determined in geocentric

coordinates with only the knowledge of time at either beginning or end of exposure, even with no timing marks on either satellite trace. Of course, timing marks on either or both traces are necessary to project this path to the celestial sphere or to locate the satellite on the path as a function of time.

In effect, by cancelling the Earth's rotation, right-ascension drives disable the function of the Earth's rotation as a master clock which

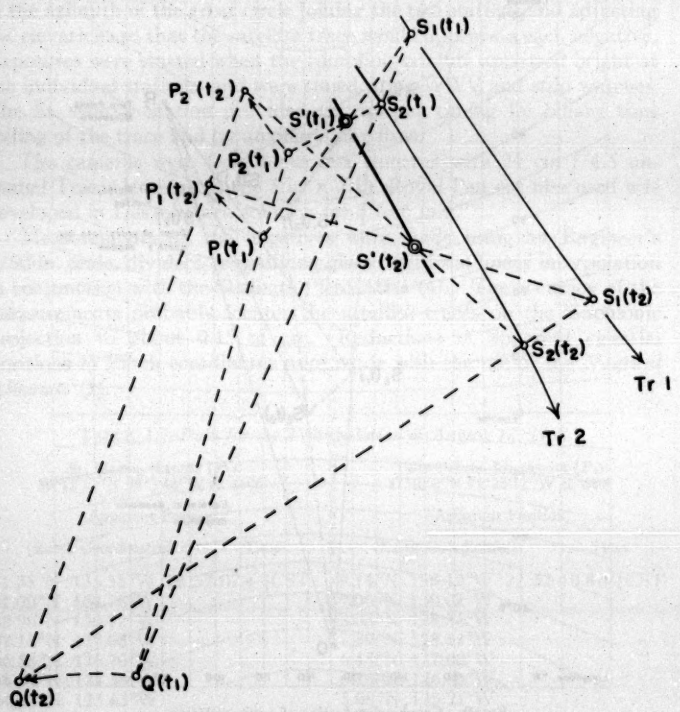

FIG. 8. Gnomonic projection of right-ascension drive camera data.

places ubiquitous timing markers on the fixed camera photographs. Since the use of right-ascension drives requires the re-insertion of timing data normally provided by the Earth's rotation, their use results in more cumbersome time keeping and heavy penalties in case some of the timing is lost.

A third application to theory is found in the selection of pairs of stations for observing satellites in orbits at various inclinations to the equatorial plane. For certain paths the construction to locate the point pairs and the sub-satellite point is indeterminate. For these paths the two traces coincide on the superimposed triangulation

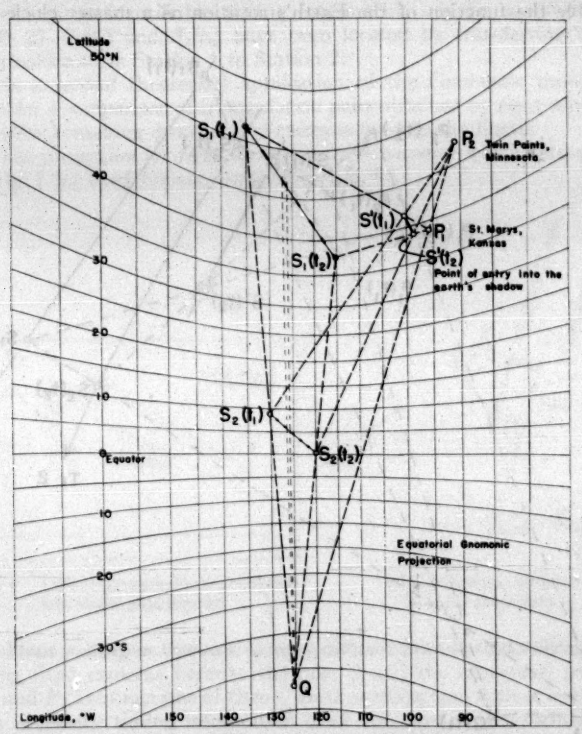

FIG. 9. Graphical reduction of a triangulation.

photographs, and the parallax can be determined only from accurate, identifiable timing markers present on both traces. This situation, which demands elaborate and expensive shutters and timing gear, can be avoided by choosing observers' stations on a great circle as nearly perpendicular to the sub-satellite path as possible.

GRAPHICAL REDUCTION OF A PHOTOGRAPHIC TRIANGULATION OF 1958-δ-1

Figure 9 is an application of an equatorial Gnomonic projection to obtain data from a triangulation pair of photographs taken August 12, 1958, from stations about 750 miles apart.

One station was at St. Marys, Kansas, and the other at Twin Points, north of Two Harbors, Minnesota. The photographs were obtained without inter-station communication by training the cameras in the azimuth of the great circle joining the two stations and adjusting the elevations so that the satellite trace would appear on each negative. Exposures were started when the tumbling satellite appeared bright at the individual stations, and were timed using WWV and stop watches. The St. Marys Station provided the master timing by binary time coding of the trace and by auxiliary star fixes.

The cameras were fixed focus box cameras with 21 cm $f/4.5$ uncoated Tessar lenses, and the 5 in × 7 in. Royal Pan cut film used was developed in Dektol stock solution for 5 to 6 min.

Measurements on the negatives were made using an Engineer's 1/50-in. scale, dividers, magnifying glass, and local linear interpolation in conjunction with the Skalnate Pleso Atlas (4). The accuracy of the measurements probably locates the satellite traces on the Gnomonic projection to about 0.1° of arc. Reductions of apparent celestial positions to Earth coordinates were made with the use of the *Nautical Almanac* (5).

TABLE I.—*Data for the Triangulation of August 12, 1958.*

St. Marys, Kansas (P_1) 39°12′12″N 96°3′48″W el. 1100′			Twin Points, Minnesota (P_2) 47°10′0″N 91°25′12″W el. 620′		
Apparent Position (Earth Coordinates)		Time	Apparent Position (Earth Coordinates)		Time
51.35°N	135.55°W	22:52:07±¼CST	9.14°N	130.19°W	22:52:10.8±¼CST
51.00°N	134.76°W		9.06°N	130.02°W	
48.90°N	130.72°W		8.03°N	128.84°W	
47.13°N	127.68°W		7.89°N	128.63°W	
46.53°N	126.70°W		6.45°N	127.00°W	
44.94°N	124.38°W		6.29°N	126.80°W	
44.48°N	123.43°W		5.00°N	125.21°W	
42.77°N	121.44°W		4.75°N	125.01°W	
42.00°N	120.40°W		3.58°N	123.49°W	
40.63°N	118.81°W		3.35°N	123.32°W	
40.00°N	118.10°W		2.30°N	122.07°W	
39.86°N	117.90°W		1.93°N	121.51°W	22:53:12±6CST*
38.20°N	116.00°W				
37.94°N	115.65°W	22:53:08±2CST*			

* Apparent entry into Earth's shadow.

The data obtained by these methods,[3] along with the observers' positions, are given in Table I, and plotted in Fig. 9.

The figure displays the projected geometry of triangulation, showing the relationships between the apparent and absolute paths of the satellite, and correspondences between apparent points of entry into the Earth's shadow and points at which the satellite appeared bright to the individual stations.

The data in Table II were obtained, after point-pairing on the Gnomonic projection by the use of Eqs. 5 and 1.

The residual E, which is a measure of the over-all accuracy attained, is considered satisfactory, and may even seem remarkable in view of the methods used, since errors occur in locating the observers accurately, reading positions on the negatives, in the linear interpolation used on

TABLE II.—*Data Reduced from the Triangulation Pair of August 12, 1958.*

Apparent Position Pairs as Read from Fig. 9, with Q at 42°9′53″S 125°33′4″W

Point	Time, CST	Apparent Position	
$S_2(t_1)$	22:52:07±¼	9°25′50″N	130°20′5″W
$S_1(t_1)$	22:52:07±¼	51°21′0″N	135°33′6″W
$S_2(t_2)$	22:53:08±2	0°42′15″N	120°4′14″W
$S_1(t_2)$	22:53:08±2	37°56′24″N	115°39′18″W

Point	Time, CST	True Positions Position		Hgt., mi.*	E(residual)**
$S'(t_1)$	22:52:07±¼	41°44′21″N	100°42′58″W	618.75	2.002 mi.
$S'(t_2)$	22:53:08±2	39°12′17″N	98°53′20″W	648.85	1.347 mi.
Inclination of orbit		66°12′29″±12′			

* Above the Hayford spheroid (7).
** See Eq. 7.

the Skalnate-Pleso charts, in the point pairing on the Gnomonic projection, since no correction was made to reduce geodetic coordinates to geocentric ones, and since one negative was slightly out of focus, and the other exhibited evidence of camera movement.

Figure 9 shows, however, that errors in apparent positions are considerably reduced when the true positions are obtained, essentially because of the large base line used for the triangulation and the favorable orientation of the satellite's orbit.

Although the Gnomonic projection is a very convenient method for displaying the geometry of triangulation for visualization and a graphic check of the analytic solution, the pairing of points, for which it has been used in the triangulation reduced here, can be carried out analytically to obtain a better pairing by choosing points which minimize E.

[3] The authors wish to thank Lt. Col. J. C. Minahan and other men of the 90th A and E Squadron of Forbes AFB for their help and loan of equipment in obtaining a large number of triangulation pairs of 1958-8-1 which have yet to be reduced.

In Fig. 9 it can be seen that the points which correspond to the satellite's entry into the earth's shadow were not seen at both stations. This is because the satellite does not disappear suddenly, but gradually fades out as it enters the Earth's shadow because the shadow does not have a sharp edge. The faint termination of the trace on the St. Marys, Kansas negative is missing altogether from the Twin Points, Minnesota negative because of the increased distance of the satellite from this observer.

Thus, although single pictures showing traces entering or leaving the Earth's shadow yield rapid information concerning the height and position of the orbit (6), their accuracy is inherently limited.

Correspondences between brightnesses of the satellite as observed from the two stations are also evident in the figure. These correspondences can be obtained from triangulation photographs without timing marks on either satellite trace. If the satellite is assumed to be cylindrical, these flashes yield some information concerning its orientation. If it appears bright to one observer, it can be inferred that the satellite is perpendicular to the bisector of the angle formed at the satellite by the observer and the sun. Consequently, if it appears bright simultaneously at two different stations, it is perpendicular to the plane containing the sun and the two stations. The fact that a satellite appears bright to one station and dark to another station simultaneously serves only to eliminate certain satellite orientations. Since most observations will be of this type, it is clear that an extraordinary amount of data would be required to determine the tumbling motion of a satellite uniquely, even if it were cylindrical.

PSEUDO-TRIANGULATION

Two observations on the same satellite made an integral number of revolutions apart by one or two observers allow a scheme of data reduction which might be called "pseudo-triangulation." Two assumptions are necessary, namely, that (1) the rate of orbit precession is well known, and (2) the changes in orbit parameters, such as the change in average height and the perigee position, are negligible during the time between the two given revolutions of the satellite. Both assumptions are reasonable except, possibly, during the final week or two of a satellite's life.

Figure 10 represents without loss of generality, the simplified geometry of pseudo-triangulation. The Earth is viewed from the North polar axis with the initial observer and satellite positions at P_1 and S_1, respectively. The basis for pseudo-triangulation stems from the essentially constant rate of the Earth's rotation about its polar axis. N-satellite revolutions later, the observer is transported to a new position P_2 from where a second observation is made on the same portion of the orbit. The second position differs from the first only in the

"right-ascension" angle which can be calculated if the times of the two observations are known. In the meantime, the satellite has precessed to S_2 by some precession angle $\Delta\Omega$ which is assumed to be known. The geometry clearly shows that correction for the precession angle can be made by simply subtracting the precession angle from the "right-ascension" angles of both P_2 and the apparent position measurements from P_2. In effect, the pseudo-triangulation is performed from positions P_1 and P_2', using apparent position data corrected for precession. The data can be reduced by either of the two schemes previously introduced.

It is clear that two observers as well as one can perform pseudo-triangulation since, in either case, their effective station positions can

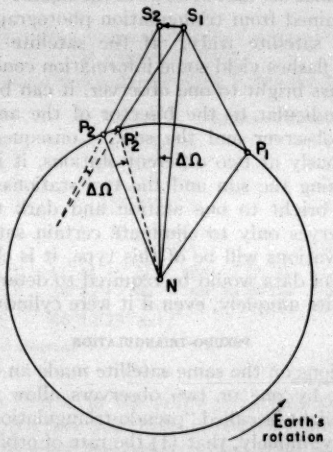

FIG. 10. Correction for precession in pseudo-triangulation.

be calculated with equal ease. This allows interesting possibilities of pseudo-triangulation by two observers located too remotely for normal triangulation.

It is desirable to keep the number of satellite revolutions between the two observations a minimum in order to keep the uncertainties introduced by the assumptions small.

Unlike normal triangulation, point pairing cannot be done by timing information alone since it is doubtful that time predictions will ever be sufficiently accurate for this purpose. This fact makes the analytic solution somewhat more cumbersome than the graphical method for pseudo-triangulation.

CONCLUDING REMARKS

Photographic triangulation, as described herein, is potentially the most accurate method now known for tracking satellites brighter than fourth or fifth visual magnitude. With good optics and timing apparatus, accuracies of the order of 1″ of arc are possible. This is sufficient to locate a satellite of moderate height to well within 100 yards. With judicious orientation of triangulation stations, relatively simple equipment can be used since fixed cameras are preferred for triangulation.

Triangulation, however, is not limited to data obtained photographically. Any method of observation which obtains azimuths and elevations which overlap in time from two stations yields data suitable for triangulation. Thus, pairs of Mini-Track stations could be used for triangulation purposes.

Triangulation possesses three unique advantages:

First, the method is absolute. As triangulation does not depend on any kinetic theory, it can be used to track objects moving in arbitrary paths. Applications to tracking of guided missiles or aircraft are evident.

Secondly, since it can provide vector velocity and position without *a priori* assumptions on orbit, it can be used to determine the initial conditions of a newly launched satellite from which the initial orbit determination can be obtained from kinematic theories. With digital computer programming, it is possible to accomplish this before the satellite completes its first revolution.

Thirdly, two or more stations can check their accuracies against each other and obtain an indication of the order of error in the final results by taking advantage of the redundancy of data.

Pseudo-triangulation, while not as accurate as triangulation and applicable only to satellites, has the advantage that it can be carried out by one station on either successive passes or successive days, and with a minimum of personnel.

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SOLUTION TO THE REALIZABILITY PROBLEM FOR IRREDUNDANT BOOLEAN BRANCH-NETWORKS *

BY

LARS LOFGREN [1]

SUMMARY

The existence problem for Boolean branch-networks (contact-networks) with only one branch for each different literal of the corresponding Boolean form is a recognized key-problem for the synthesis of combinatorial (nonoriented) switching circuits. It is equivalent to the topological problem of finding a necessary and sufficient condition for a matrix of integers mod 2 to be a circuit matrix (incidence matrix of a loop-graph). Nor has a simple solution for this problem been given so far. This type of network, called irredundant, is defined for the 2- and n-terminal case and a simple general solution is given to the existence and realizability problem. It is also a solution to the above-mentioned topological problem. This problem is also connected with ordinary circuit theory. The theory of non-oriented networks is readily extended to irredundant networks of oriented branches.

I. INTRODUCTION

Let $B(a, a', b, b', \cdots)$ be a Boolean function of a finite number of binary variables (literals) a, a', b, b', $\cdots$. A literal is a letter either affirmed or negated. Throughout this paper we will assume that B is transformed so as to contain no redundant literals. For such a transformation we refer to the works of Quine (9, 10).[2]

We say by definition that an irredundant branch-network (or contact-network) of B is a B-generating network with a 1–1 correspondence between its (non-oriented) branches and the irredundant literals of B. The expression B-generating (or a network "of" B) will be made precise in Sec. II so that an eventual irredundant network is always referred to a specified form of B. We may specify B also with vanishing intersections (containing a letter both affirmed and negated).

The main problem in deciding whether a Boolean form has a network with only one branch per literal is how to decide whether there exists a graph to a prescribed loop-set matrix that corresponds to the Boolean form. In connection with switching circuits this matrix-graph problem has been raised by Okada (8) and Seshu (11, 12). It is also an actual problem in ordinary circuit theory (11, 12, 1, 4).

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Attempts to solve it have been made by Gould (2, 3) and Guillemin (4), but to the author's knowledge no effective solution has been given so far. In this connection a paper by Whitney (17) should be mentioned, although it is not specifically directed towards a solution to this problem. Also a paper by Lund (7) should be mentioned.

The main purpose of this paper is to give two conditions on a Boolean form B which together are necessary and sufficient for the existence of an irredundant network of B. The main condition, the subrearrangement theorem of Sec. III (together with the path-formation method of Sec. IV) is an effective constructive solution to the mentioned matrix-graph problem.

In Sec. V we will see that the theory of irredundant two-terminal networks (a network generating one Boolean function) is readily extended to the n-terminal case (a network generating a system of Boolean functions).

In Sec. VI we will make a brief extension of the theory of non-oriented networks so as to cover also networks with oriented branches.

The irredundant networks play an important role for the general minimum problem in combinatorial switching circuits and even more for the problem of how to design a switching network with a minimum number of redundancy-branches[3] for a certain protection against branch errors (5, 6).

II. PROPERTIES OF THE GENERATOR SETS Γ_L AND Γ_H

Let us consider a graph of a two-terminal branch-network according to Fig. 1. The terminal branch T is connected to the two vertices of

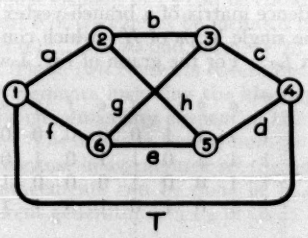


FIG. 1. Graph of a two-terminal network.

[3] A paper on the subject is being prepared for publication. The main result of this forthcoming paper is that the minimum number of branches n in a redundancy-network of contact type generating a Boolean function B also if there are errors in any w branches, is bounded by:

$$m(w+1)^2 \leq n \leq M(w+1)^2.$$

M is the minimum number of branches in a network for the generation of B without any prescribed error correction and m is the number of irredundant literals of B. So for functions which have irredundant networks, the two bounds coincide. For other functions n approaches the lower limit when w is increased.

the network between which the Boolean function B is generated. T corresponds to B. The proper switch branches are denoted with the corresponding literals of B. We will use the convention: if a literal has the value 1, the corresponding switch branch is in "make" condition and if it has the value 0 the branch is in "break" condition. Thus "make" or "break" between the endpoints of two series-connected branches a and b is represented by the intersection (conjunction, "product") ab and for a parallel connection we obtain the union (disjunction, "sum") $a \cup b$. If a dual graph exists, it represents the network of the opposite convention (compare Shannon (13, 14)).

The graph of Fig. 1 is uniquely represented with the incidence matrix $H_T{}^2$ (see Veblen (15)):

$$H_T{}^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccccc} T & a & b & c & d & e & f & g & h \end{array} \\ \left\| \begin{array}{cccccccc} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array} \right\| \end{array}.$$

A branch-vertex incidence matrix $H^2{}_T = \|\eta_{ij}\|$ of a non-oriented graph of c branches (including the T-element) and r vertices has c branch columns and r vertex rows. η_{ij} is 1 if the i^{th} vertex is incident with the j^{th} branch and 0 otherwise. Any matrix whose elements are 0's and 1's with the property that each column has precisely two 1's can be regarded as the incidence matrix of a branch-vertex graph.

Let us denote the single loops of $H^2{}_T$ which contain the T-element with the loop matrix L_T. For the graph of Fig. 1 we have:

$$L_T = \begin{array}{c} \begin{array}{cccccccc} T & a & b & c & d & e & f & g & h \end{array} \\ \left\| \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ & & & & \vdots & & & & \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{array} \right\| \end{array}.$$

Each row of L_T represents a single loop and the 1's in a row indicate the branches with which it is incident.

For the matrix product $H^2{}_T \bar{L}_T$, where $\bar{L}_T$ is the transpose of L_T, we have

$$H^2{}_T \cdot \bar{L}_T = 0 \tag{1}$$

(compare (8, 15)). The matrix product (row-column rule) is defined

with ordinary multiplication and with mod 2 addition. 0 is a matrix with 0's in all positions.

The Boolean function of a specified network (or H^2_T) is determined from the loop matrix L_T. A conjunction of the function is the intersection of those literals $(a, b, \cdots)$ of a row which are marked with 1. The function is a union (disjunction) of all these intersections (conjunctions, rows of L_T).

Let us consider the complete solution H_T of (1) for a specified L_T. It is evidently a group G_H under addition mod 2. For if α and β are two solution elements, so is $\alpha + \beta$ mod 2. Further, the identity element of G_H is a string of 0's (which obviously also is a solution of (1)). The inverse of an element of G_H evidently is the element itself. Finally since the operation of addition mod 2 (digit by digit) is commutative, G_H is an abelian group.

If we instead consider H^2_T of (1) known and solve for L_T the complete solution G_L is similarily seen to be an abelian group under addition mod 2.

The two groups G_H and G_L are conveniently specified by generator sets Γ_H and Γ_L. The group G_H contains 2^{r-1} elements. As before, r is the number of non-zero rows of H^2_T or the number of vertices of the connected graph H^2_T. Any $r - 1$ independent (mod 2) elements of G_H form a generator set Γ_H. The number of ways in which $r - 1$ independent elements can be chosen from G_H is:

$$(2^{r-1} - 2^0)(2^{r-1} - 2^1)\cdots(2^{r-1} - 2^{r-2}).$$

This is because the first element of Γ_H can be chosen in $(2^{r-1} - 1)$ ways (the identity element 0 of G_H cannot be included in a non-trivial set of independent elements). The second element of Γ_H can be chosen in $2^{r-1} - 2$ ways. These two elements determine a subgroup of order 2^2 (that is, with 2^2 elements including the identity element). Thus a third element can be chosen as any element of the remaining $2^{r-1} - 2^2$ elements and so on. The above indicated number is, however, not the number of Γ_H sets, because in a generator set the rows can be arbitrarily rearranged. Rearrangements can be done in $(r - 1)!$ ways. Hence the number of different generator sets Γ_H of G_H is $N(r)$:

$$N(r) = (2^{r-1} - 2^0)(2^{r-1} - 2^1)\cdots(2^{r-1} - 2^{r-2})/(r - 1)! \qquad (2)$$

In the same way we define Γ_L to be a complete set of independent elements of G_L. Since H^2_T (having c columns and r rows) must be connected the number of elements of Γ_L is:

$$\mu = c - r + 1. \qquad (3)$$

μ is the so-called cyclomatic number (15).

Let us transform a Γ_L set with row replacements (one row is added mod 2 to another) so as to contain a single element γ with a 1 in the T-position and a rest, the set Γ. Each element of Γ has a 0 in the T-position. This partition

$$\Gamma_L = \gamma \cup \Gamma \tag{4}$$

is always possible since Γ_L is a generator set. Γ generates the subgroup g of G_L, and we can divide G_L into the subgroup g and the co-set c:

$$c = \gamma + g \qquad \text{mod } 2. \tag{5}$$

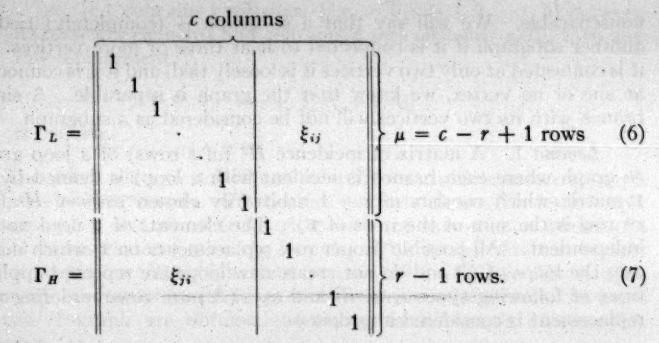
The co-set c, which is uniquely derived from a specified Boolean form B (B implies L_T and L_T implies c), consists of all the elements of G_L which have a 1 in the T-position. If an $H^2{}_T$-graph of B really exists, this graph also implies c from which we can derive a Boolean function $B(c)$ as was outlined in the above. Hence it is necessary that $B(c)$ "implies" B in the sense that if the literals of a clause of $B(c)$ are given such values that the clause assumes the value 1, then for these values B must be 1 under all circumstances. If an intersection of $B(c)$ always has the value 0 it does not violate the implication. We will call this the c-criterion:

c-criterion: A necessary condition for a Boolean form B to have an irredundant network is that the function $B(c)$, which corresponds to the co-set c (5) of $L_T(B)$, implies B.

It is not necessary to state that B shall imply $B(c)$ for it always does. Evidently c consists of any sum mod 2 of an odd number of elements of L_T. In this form the criterion seems to have been first derived by the late Dr. Gould (2). The choice of the above form is convenient for a generalization to n-terminal networks.

We are now able to define precisely the meaning of an irredundant network of a form B. We mean with this notation a network which generates the $B(c)$ function provided $B(c)$ implies B. Suppose that another network generates $B(c^*)$ where $c^* \neq c$. It can still happen that $B(c^*)$ also implies B. However we will not refer this network to the form B but instead to the form B^* which generates c^*.

Γ_L can always be transformed with row-replacement operations into an explicitly independent form, that is, a form with μ (3) columns only containing a single 1 so that not two of these 1's appear in the same row. For convenience the columns are rearranged (if necessary) so that the diagonal from (6) is obtained.



It is easily seen that the explicitly independent Γ_H-form (7) satisfies

$$\Gamma_H \overline{\Gamma}_L = 0 \tag{8}$$

(compare (1)) if the μ rows of the right part of (6) are the columns of the left part of Γ_H.

From the results of Veblen (15) it follows that $H_T{}^2$ of a connected graph consists of a generator set Γ_H and of one dependent row, which is the sum mod 2 of the rows of Γ_H.

It is easily seen that any of the $N(r)$ (2) generator sets can be obtained by repeated row-replacement operations on a specific generator set Γ_H. Now we have reached the main problem of this paper: is it possible to decide whether a generator set Γ_H can be transformed into Γ_{H^2}-form (having at most two 1's in each column) without investigating all possible row replacements (or without another large number of investigations)? The answer is definitely yes and the method is given by the subrearrangement theorem of the next section.

III. THE SUBREARRANGEMENT THEOREM

We will first derive a subrearrangement criterion. The idea behind this criterion is to derive a subproblem with known solutions, which will give character to the main realizability problem. We will see that it is possible to give the subproblem essentially the same nature as that of the main problem so that the number of investigations for the main problem will be limited to those which correspond to the transformations of one solution of the subproblem to another.

For the subproblem we must admit separable loop graphs as solutions. In the following lemma for the subrearrangement criterion, we will therefore speak of H^2-graphs for subproblems as if they could be separable. An $H^2{}_T$-graph for the main problem is however always

nonseparable. We will say that a subgraph is (completely) tied to another subgraph if it is connected to it at three or more vertices. If it is connected at only two vertices it is loosely tied, and if it is connected at one or no vertex, we know that the graph is separable. A single branch with its two vertices will not be considered as a subgraph.

Lemma 1. A matrix of incidence H^2 (of r rows) of a loop graph (a graph where each branch is incident with a loop) is defined by its Γ-matrix which consists of $r-1$ arbitrarily chosen rows of H^2 (the r^{th} row is the sum of the rows of Γ). The elements of Γ need not be independent. All possible proper row replacements on Γ which maintain the loops of H^2 and do not create new loops are repeated applications of following types *i*, *ii*, *iii* and *iv*. (A pure row-reordering row replacement is considered improper.)

i. If H^2 contains loosely tied subgraphs, say H_{ij} which has only the two vertices r_i and r_j in common with another subgraph, then the row replacement:

$$r_i \to r_i + \sum_{H_{ij}}^{\text{exc } r_i, r_j} (r_\nu), \qquad r_j \to r_j + \sum_{H_{ij}}^{\text{exc } r_i, r_j} (r_\nu)$$

is possible. (r_i is replaced by the sum mod 2 of itself and all the vertices of H_{ij} except for r_i and r_j and correspondingly for r_j.) This operation corresponds to a change of connectivity of the two-port H_{ij} so that the two-port is turned around at its two connection vertices.

ii. If H^2 is separable a cut vertex r_{ij} (the only vertex incident with both of the two subgraphs H_i and H_j) can be changed into another cut vertex r_{kl}, which is formed when r_{ij} is split into r_i and r_j and when a vertex r_k of H_i and a vertex r_l of H_j coalesce:

$$r_k \to r_k + r_l = r_{kl},$$

$$r_{ij} \to r_{ij} + \sum_{H_j}^{\text{exc } r_{ij}} (r_\nu) = r_i, \qquad r_l \to r_l + \sum_{H_j}^{\text{exc } r_l, r_{ij}} (r_\nu) = r_j.$$

iii α. If H^2 consists of disconnected subgraphs, say H_k and H_l, these subgraphs can be joined at a single vertex r_{kl}, which is formed when a vertex r_k of H_k and a vertex r_l of H_l coalesce:

$$r_k \to r_k + r_l = r_{kl}, \qquad r_l \to \sum_{H_l} (r_\nu) = r(0).$$

iii β. If H^2 is separable and has, besides a cut vertex r_{ij} which joins the subgraphs H_i and H_j, also a zero-row $r(0)$, then the cut vertex can be split:

$$r(0) \to r(0) + \sum_{H^i}^{\text{exc } r_{ij}} (r_\nu) = r_i, \qquad r_{ij} \to r_{ij} + r_i = r_j.$$

iv. If Γ contains k relations $\sum_i r_\nu = 0 (i = 1, 2, \cdots, k)$ between its non-zero elements and contains n zero elements $r_\nu(0)$, then any row replacement:

$$r_j \to r_j + \sum_i r_\nu + r_\nu(0)$$

is possible.

The *proof* will run as follows. An H^2-matrix determines a graph uniquely and conversely. H^2 specifies uniquely a complete set of loops, the group G_L, and all other matrices with the same G_L (or with loops in 1–1 correspondence with the loops of H^2) must be obtained by row replacements on H^2 which conserve the H^2-form (two 1's in each column). All graphs with loops in 1–1 correspondence with the loops of the H^2-graph are obtained according to Whitney's 2-isomorphic theorem (16):

> If there is a 1–1 correspondence between the branches of the two graphs G and G' so that loops correspond to loops, then the graphs are strictly 2-isomorphic.

Two graphs G and G' are 2-isomorphic if one can be transformed into the other by the connectivity changes:

1. If $G = H_1 + H_2$, where H_1 and H_2 have just the vertices r_i and r_j in common and these vertices are connected in both H_1 and H_2, then H_1 is turned around at these vertices.
2. Break a graph at a single vertex into two connected pieces, or join two pieces (not connected to each other) at a single vertex.

Finally there is only one proper row replacement for each change of connectivity if Γ is a generator set. These are easily found to be the ones listed above in Cases *i* and *ii*. However if Γ has k relations it consists of $(k + 1)$ disconnected subgraphs (compare Veblen (15)). If two disconnected pieces are connected at a single vertex a zero row is created. Conversely a cut vertex may be split at the expense of a zero row. Corresponding row replacements are easily found to be the ones indicated under Case *iii*. These are however not unique because of the relations and zero rows. For let us represent a row replacement by a square transformation matrix T of $r - 1$ rows. Suppose a certain change of connectivity transforms the matrix Γ to Γ^*:

$$T\Gamma = \Gamma^*. \tag{9}$$

Suppose that there exists another matrix T' for this transformation. Then

$$(T + T')\Gamma = 0 \quad \text{(sum mod 2)}. \tag{10}$$

If Γ is a generator set the only solution is $T' = T$. However if Γ has k dependent elements and n zero rows we find that T and T' may differ in $k + n$ columns, which means that also the replacements *iv* are possible.

Before stating the subrearrangement criterion we will define a $\nu' - \nu''$ partitioning of Γ_H and Γ_L. Let ν' be a number of columns of Γ_H, each having at most two 1's. The remaining $\nu'' = c - \nu'$ columns may each have more than two 1's. A corresponding Γ_L-matrix is transformed with row-replacement operations so that $(\Gamma_L)_{\nu''}$, that is, the ν'' columns of Γ_L, are converted into independent rows and zero rows. The rows of the so-transformed Γ_L-matrix which contain these independent rows of $(\Gamma_L)_{\nu''}$ are deleted, and the remaining rows are denoted Γ_L'. All non-empty columns, say ν', of $(\Gamma_L')_{\nu'}$, are denoted $(\Gamma_L')_{\nu'}$. The same columns of Γ_H are denoted $(\Gamma_H)_{\nu'}$. A column which is not a ν'-column is a ν''-column ($\nu'' = c - \nu'$).

The subrearrangement $H^2{}_T$-criterion: The only row replacements which may convert Γ_H into Γ_{H^2}-form are the replacements *i*, *ii*, *iii*, *iv* of Lemma 1 where Γ is a ν'-partition of $\Gamma_H/(\Gamma = (\Gamma_H)_{\nu'})$. If the r^{th} row of $(H^2)_{\nu'}$, which is not present in $(\Gamma_H)_{\nu'}$, is involved in a row replacement, a row r_i which is not involved, is replaced by the r^{th}. The $\nu' - \nu''$ partitions are unaffected by this replacement. Should there be an empty column of Γ_H, no Γ_{H^2} form of Γ_H exists.

Let us in the *proof* first consider the case when there is an empty column of Γ_H. Then any other form of Γ_H must also have only 0's in this column, and we know that no $H^2{}_T$-form can exist. Next we will prove that $(\Gamma_H)_{\nu'}$ is a Γ-set of an H^2-matrix (see Lemma 1) corresponding to a loop graph with all its loops in 1–1 correspondence with those loops of H_T which are only incident with the ν'-branches. Because $(\Gamma_H)_{\nu'}$ has exactly as many rows as Γ_H, and because a row replacement on $(\Gamma_H)_{\nu'}$ operates in the same way on Γ_H it then follows that the row replacements according to Lemma 1 are the only ones which can possibly convert Γ_H into Γ_{H^2}-form. So let us denote with G_L' the group which is generated by Γ_L'. The set containing the remaining elements of G_L is denoted G_L''. The ν'-columns of G_L' and G_L'' are denoted $(G_L')_{\nu'}$ and $(G_L'')_{\nu'}$. In the same way for the ν''-partitions. Equation 1 is with these notations equivalent to:

$$\left.\begin{aligned} \Gamma_H \overline{G_L'} &= 0 \\ \Gamma_H \overline{G_L''} &= 0 \end{aligned}\right\} \qquad \begin{matrix} (11) \\ (12) \end{matrix}$$

Since $(G_L')_{\nu''}$ has only zero-elements, (11) implies

$$(\Gamma_H)_{\nu'}(\overline{G_L'})_{\nu'} = 0 \qquad (13)$$

and from (12) we obtain:

$$(\Gamma_H)_{\nu''}(\overline{G_L''})_{\nu''} = (\Gamma_H)_{\nu'}(\overline{G_L''})_{\nu'}. \tag{14}$$

Hence, when we want to determine Γ_H from (1), this is equivalent to a determination of the $(\Gamma_H)_{\nu''}$-part from (13) only, and the remaining $(\Gamma_H)_{\nu''}$-part is then determined from (14). So we have referred the determination of $(\Gamma_H)_{\nu'}$ to a problem of the same character as the problem of determining H_T[2] from (1) with a specified group G_L. We know that $(G_L')_{\nu'}$ is a proper loop group, because it consists of all those loops of G_L which are only incident with the ν'-branches, and it contains no zero column. $(H_T)_{\nu'}$ is obtained by adjoining an r^{th} row to $(\Gamma_H)_{\nu'}$ and we know that it is of H^2-form. Hence the criterion is proved.

When trying to transform a Γ_H-matrix into Γ_{H^2}-form, this criterion can always be applied and is an effective short cut, although some trial and error is involved. We will therefore only use it in deriving the subrearrangement theorem. This theorem is the desired solution to our problem. It is very easy to apply because it is systematic and only contains a few (μ) applications of a single operation. ($\mu = c - r + 1$ is as before the cyclomatic number, that is, the number of loops of Γ_L.) For the proof of the theorem we also need the following lemma.

Lemma 2. Each of the $\mu = c - r + 1$ rows of an explicitly independent generator set Γ_L represents a single loop in an eventual irredundant network for L_T.

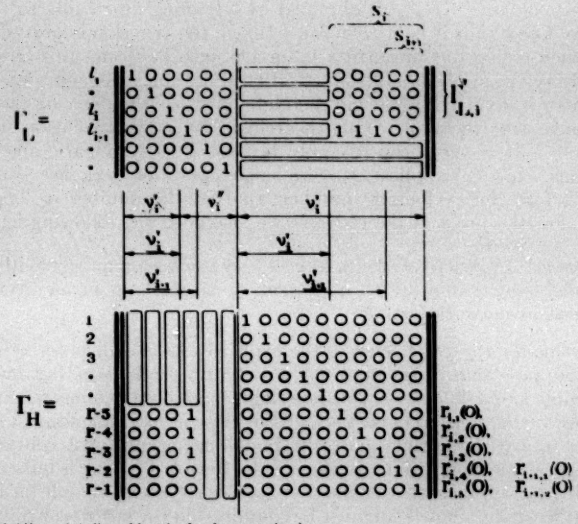
Let us for the *proof* refer to the explicitly independent form (6) of Γ_L. Suppose that a row does not represent a single loop, but instead a multiple loop (that is, two or more single loops either connected at one or more vertices or disconnected). If in this case a single branch is removed from the multiple loop, the remaining graph must contain at least one single loop. Since in each of the rows of Γ_L there is only one 1 in the first μ columns, this means that if a row represents a multiple loop, then at least one single loop must be contained in the last $r - 1$ columns of that row. But this cannot happen, for Γ_L is a generator set of G_L which represents all loops (single and multiple), and any element of G_L must contain at least one 1 in the first μ columns.

Let us now state the theorem.

The subrearrangement theorem: A necessary and sufficient condition that a loop generator set Γ_L—in an explicitly independent form—has a pair Γ_H of Γ_{H^2}-form, is that a connected subgraph which corresponds to some of the loops (rows) of Γ_L can be *i*-, *ii*-rearranged (see Lemma 1), so that the branches of the subgraph which are contained in another loop of Γ_L form a single path. The connected subgraph which also contains this new loop is obtained

by connecting to the two end vertices of the path, the other branches of the new loop, connected in series. If no branch of the loop is contained in the previous subgraph, the new loop is connected to the subgraph at one arbitrary vertex. If the process is started with a single loop of Γ_L as subgraph, it ends up with a desired H^2_T-graph, or if a path-formation with operations *i* or *ii* is impossible, then no H^2_T-graph exists. The order in which the loops are taken is arbitrary. Also if more than one path-formation is possible, it is without significance which one is chosen.

Let us for the *proof* consider a Γ_L, Γ_H-pair:



(15)

(16)

(A boldface ν is indicated by $\underline{\nu}$ in the above matrices.)

Γ_L is transformed into an explicitly independent form, that is, with μ different columns only containing one 1. The corresponding (compare (8)) pair Γ_H (16) is then immediately obtained in explicitly independent form. The rows of the right part of Γ_L are the columns to the left of Γ_H.

We will temporarily assume that it is possible to construct a loop graph corresponding to some of the rows of Γ_L, say the i first (in (15) the three first). Let the $\mu - i$ columns of the independent part of Γ_L which have 1's in the remaining $\mu - i$ loops of Γ_L be the ν''-partition of the Γ_{H^2}-criterion. We denote it with ν_i'' and we use the index i also for the ν'-partition ν_i'. There may be a number of columns of $(\Gamma_L')_{\nu_i'}$ which are empty, say the s_i last. Withdrawing them from ν_i' we have

the ν_i'-partition as indicated in (15). The $(\Gamma_H)_{\nu_i'}$-matrix consists of s_i zero rows $r_{i,1}(0)$, $r_{i,2}(0)$, $\cdots$, and the other rows are explicitly independent. According to our assumption, $(\Gamma_H)_{\nu_i'}$ can be transformed into Γ_{H^2}-form. There are two possibilities for the corresponding subgraph. Either the graph G_i of the independent rows is nonseparable (Fig. 2*a*) or it is separable (Fig. 2*b*). If, for instance, in the last case there are s_i cut vertices and s_i zero rows, the graph can be split (*iiiβ* of Lemma 1) into $s_i + 1$ disconnected pieces (Fig. 2*c*). In order to diminish the number of types of rearrangements, we can always require that G_i shall consist of only one piece. Even if G_i is separable and consists of disconnected pieces, we know that the final H_{T^2}-graph shall be connected, but we do not know *a priori* how the pieces shall be connected. But if we connect them arbitrarily (*iiia*), we can always change their connectivity in any desirable way only with the operation *ii* of Lemma 1.

Let us next consider the $(\Gamma_H)_{\nu'_{i+1}}$-matrix, which corresponds to the same single loops as before and one further, say the $(i+1)^{\text{th}}$ loop

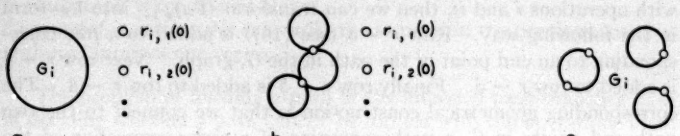


FIG. 2. Possibilities for an H^2-graph.

l_{i+1} of Γ_L. The ν_{i+1}'-partition is indicated in (15) and (16). There are two kinds of possibilities:

1. The s_i-columns to the right in Γ_L are still empty in the first $i+1$ rows. Then $(\Gamma_H)_{\nu'_{i+1}}$ has again s_i zero rows, and the other rows are independent.

2. One or more $(s_i - s_{i+1})$ of the s_i-columns are non-empty in the first $i+1$ rows of Γ_L. The $(\Gamma_H)_{\nu'_{i+1}}$-matrix has s_{i+1} zero rows and the other rows are independent. (In (15) $s_i - s_{i+1} = 3$.)

Let us first consider Case 1. We know that $(\Gamma_H)_{\nu'_i}$ has a $(\Gamma_{H^2})_{\nu'_i}$-form. $(\Gamma_H)_{\nu'_{i+1}}$ only differs from $(\Gamma_{H^2})_{\nu'_i}$ in that it contains one further column, the $(i+1)^{\text{th}}$. The content of this column is obtained by the product:

$$(\Gamma_{H^2})_{\nu'_i} \cdot \overline{(l_{i+1})}_{\nu'_i} \tag{17}$$

for we must have:

$$(\Gamma_H)_{\nu'_{i+1}} \cdot \overline{(l_{i+1})}_{\nu'_{i+1}} = 0 \tag{18}$$

(compare (13)). (17) means that the $(i+1)^{\text{th}}$ column of $(\Gamma_H)_{\nu'_{i+1}}$ has a 1 in each row that corresponds to an end vertex of a path-piece formed by the l_{i+1}-branches that are incident with the G_i-graph. It now follows from the Γ_{H^2}-criterion that a necessary and sufficient condition for $(\Gamma_H)_{\nu'_{i+1}}$ to be of Γ_{H^2}-form is that it is possible to rearrange the connected G_i-graph with operations *i* and *ii* so that the branches of it which are incident with the loop l_{i+1}, form a single path (with two end vertices). We know that the path cannot contain a loop, for then l_{i+1} would be a multiple loop which is in contradiction with Lemma 2. The G_{i+1}-graph is simply obtained by connecting the $(i+1)^{\text{th}}$ branch between the two end vertices of the path.

Let us consider Case 2 where $s_{i+1} \neq s_i$, that is, $(\Gamma_H)_{\nu'_{i+1}}$ contains $(s_i - s_{i+1})$ non-zero rows more than $(\Gamma_H)_{\nu'_i}$ and the ν'_{i+1}-partition contains $(s_i - s_{i+1} + 1)$ branches more than the ν_i'-partition. (17) gives the content of the $(i+1)^{\text{th}}$ column of $(\Gamma_H)_{\nu'_{i+1}}$, here, however, only in the positions corresponding to the non-zero rows of $(\Gamma_H)_{\nu'_i}$. Thus we see that if the path-pieces of the l_{i+1}-branches which are incident with the G_i-graph can be rearranged into a single path in G_i with operations *i* and *ii*, then we can transform $(\Gamma_H)_{\nu'_{i+1}}$ into Γ_{H^2}-form in the following way. Row $r-3$ (see (16)) is added to a row corresponding to an end point of the path in the G_i-graph. Next row $r-4$ is added to row $r-3$. Finally row $r-5$ is added to row $r-4$. The corresponding geometrical construction is that we connect to the two path end points in G_i a path consisting of a series-connection of the remaining new loop branches of l_{i+1}. The sufficient path-condition on G_i is also necessary for we know that all $(\Gamma_{H^2})_{\nu'_{i+1}}$-matrices are generated with operations *i* and *ii* of Lemma 1 from the one obtained. If the G_i-graph is separable it might be possible to rearrange the branches of the l_{i+1}-loop so that those branches which are incident with the G_i-graph form more than one path. But then the G_i-graph is not connected and it shall be according to the theorem when the G_{i+1}-graph is formed. After that, of course, any *i* or *ii* operation on G_{i+1} is allowable. Finally, if the intended path branches of G_i cannot be rearranged into a single path, then no G_{i+1}-graph exists, nor an H^2_T-graph. For we know from the Γ_{H^2}-criterion that we can determine Γ_{H^2} by determining any $(\Gamma_{H^2})_{\nu'}$-partitions successively (compare (13)), and we have shown that the above ν_i'-partitions of an explicitly independent Γ_L-set are ν'-partitions. So if no single path of the l_{i+1}-branches can be made in G_i, the only construction would be to close the l_{i+1}-branches between the path pieces in G_i (connected) so that the l_{i+1}-loop is formed. But then the number of new non-zero rows (real vertices) in (16) would be less than $s_i - s_{i+1}$, and hence this construction is impossible. This proves the theorem.

In Example 3 at the end of Sec. IV we will see what can happen if the theorem is incorrectly applied on a Γ_L which is not first given an explicitly independent form.

With the subrearrangement theorem we have referred the realizability question (the matrix-graph problem, mentioned in Sec. I) to the problem of forming a path of the intended path-branches with the operations *i* and *ii*. In the next section we shall see how this problem is easily solved.

IV. PATH-FORMATION METHOD

We will use the following expressions.

Separable subgraph: a subgraph which is connected to an internally connected subgraph at only one vertex (one for each disjoint internally connected subgraph), the cut vertex (compare Whitney (16)).
2-connected, separable subgraph: a subgraph which is connected to an internally connected subgraph at precisely two vertices (loosely tied), and which contains at least one "internal cut vertex" (a cut vertex if the subgraph is considered alone).
2-connected, nonseparable subgraph: the same as in the previous case but with no "internal cut vertex."

The problem of forming a path of the intended path-branches p_i or in deciding whether a path cannot be formed with the operations *i* and *ii* of Lemma 1 is easily solved. The necessary rearrangements can be made independent in each disjoint 2-connected, nonseparable subgraph that contains p_i-branches. After a rearrangement of a subgraph, it should be frozen, that is, not internally rearranged any more. Next a larger 2-connected, nonseparable subgraph, containing the previous "frozen" subgraph as a branch should be considered. It is individually rearranged and "frozen," and so on. So the number of steps in a path formation is essentially the number of enlargements of 2-connected, nonseparable subgraphs, containing each other in succession, necessary for a connection of the two path pieces which are most "distant" in this respect. There is no problem involved about the order in which the different path pieces shall be connected (or broken), for an enlargement of a 2-connected, nonseparable subgraph is only made towards the not contained subgraph, and the enlargement can be started with any subgraph that does not contain a smaller 2-connected, nonseparable subgraph containing some p_i-branch. If at any enlargement the required branch-arrangement (a pure series interchange of the branch order) cannot be made, the desired path cannot be formed and the final graph does not exist.

Let us first consider the case when the graph contains a number of separable subgraphs containing some intended path-branches p_i.

Since these subgraphs only can be connected at cut vertices (that is, so that no new loop is formed) it follows that the p_r-branches in a separable subgraph must be rearranged into a single path, however with no conditions on the end points, since a separable subgraph can be reconnected (*ii*) at any vertex. The subgraphs are then reconnected so that a path of the p_r-branches is formed (see Fig. 3*a* and *b*). For simplicity one could omit separable subgraphs which do not contain p_r-branches. We have now referred the investigation to a nonseparable graph.

We consider a 2-connected, nonseparable subgraph g_i, connected to the remaining graph at the two vertices v_{i1} and v_{i2} (change-around vertices) only. Even if g_i and the remaining graph contain other change-around vertices, no branch which from the beginning is not contained in g_i can be rearranged into g_i. This is an important ob-

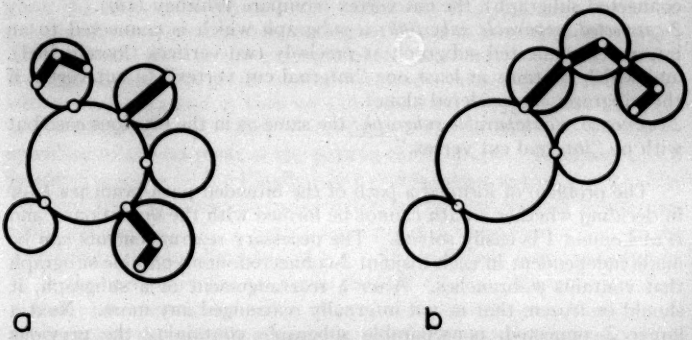


FIG. 3. Rearrangement of separable subgraphs for a path-formation.

servation on which the path-formation method is founded. If g_i does not contain a p_r-branch it can for simplicity be replaced with a single branch. If it does contain p_r-branches, these can only be brought into contact with p_r-branches outside g_i at the two change-around vertices v_{i1} and v_{i2}. Hence the p_r-branches in g_i must be rearranged so that they form paths in g_i with end points at v_{i1} or v_{i2}. We will here distinguish between two cases:

1. All p_r-branches outside g_i form a single path with its two end points at v_{i1} and v_{i2}, respectively.
2. All other cases concerning the p_r-branches outside g_i.

The case at hand is immediately recognized. For if a p_r-path incident with v_{i1} and v_{i2} exists outside g_i, then the path piece between v_{i1} and v_{i2} always exists (it cannot be broken by a rearrangement). So

if there are no more p_i-branches outside g_i than the mentioned path piece, then we have Case 1, otherwise Case 2.

In Case 2 there is only one possibility for the p_i-branches inside g_i. They must be rearranged into a single path with one end point at one of the change-around vertices (v_{i1} or v_{i2}). (Or all of them may already form a path inside g_i with one end point at v_{i1} and the other at v_{i2}.)

In Case 1 there are two possibilities. The p_i-branches inside g_i must either be rearranged into a single path with one end point at one of the change-around vertices, or they can be rearranged into two single paths, one with an end-point at v_{i1} and the other with an end-point at v_{i2}.

If not these possibilities for the two cases are at hand, then a resulting p_i-path cannot be formed.

If, however, a 2-connected, nonseparable subgraph containing p_i-branches does not exist, the whole graph must consist of a single loop (after the indicated simplification concerning subgraphs not containing p_i-branches), or the p_i-path pieces are completely tied in the graph and cannot be approached to a single path. In the case of a single loop, any pair of nonadjacent vertices is a change-around pair, but the corresponding 2-connected subgraphs contain internal cut vertices and so no 2-connected, nonseparable subgraph can exist. However, in this case it is clearly always possible to rearrange the p_i-branches so that they form a single path. For simplicity we could reduce the number of non-p_i-branches in a pure series connection to one.

The path-formation method is now simply this. Start with a 2-connected, nonseparable subgraph g_i, containing at least one p_i-branch but not containing a smaller p_i-containing 2-connected, nonseparable subgraph. (It may contain a smaller p_i-containing 2-connected, separable subgraph, however.) If Case 1, make one of the above-mentioned two rearrangements. If none of them can be made with a pure series-interchange of the branch-order, then no resulting single path exists. If it can be made, we have obtained the desired resulting path. If Case 2, make the above-mentioned rearrangement. If it cannot be made with a pure series interchange of the branch-order, then no resulting single path exists. If it can be made, the whole subgraph should be replaced with a single branch, marked path end. (If all p_i-branches in g_i already form a path with one end point at v_{i1} and the other at v_{i2}, then g_i is replaced with a single p_i-marked branch.) This procedure is then repeated either with a disjoint subgraph or with a subgraph containing the "path-end"-marked (or p_i-marked) branch of the previous step. If, in case of disjoint subgraphs, we should obtain three (or more) "path-end"-marked branches, we can at once conclude that no resulting single path can be obtained. If we, as the next step, should choose a 2-connected, nonseparable subgraph g_{i+1} containing a

"path-end"-marked branch and eventually other p_i-branches but no smaller 2-connected, nonseparable subgraph which contains p_i- (or path-end-) branches, we must be able to make rearrangements (pure series interchanges of branch orders) so that a "path-end"-marked branch really will be a path end in g_{i+1} (the path end not incident with $v_{i+1,1}$ or $v_{i+1,2}$ if more p_i-branches). If this rearrangement is not possible, no resulting single path can be formed. If possible, g_{i+1} should be replaced with a single "path-end" marked branch, and the process continued until all p_i-branches have been covered by this enlargement of the subgraphs.

We may notice that with this method there is no question about the order in which the p_i-branches shall be connected in order to form a path, for an enlargement of a subgraph is only made in order to cover the remaining graph.

Corollaries from this method are the following tests:

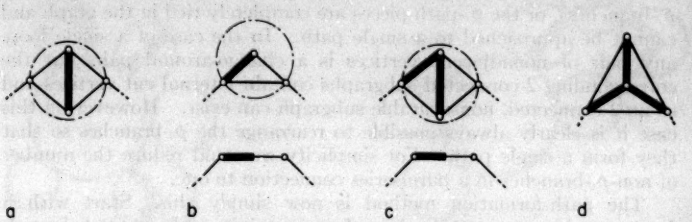


FIG. 4. Examples of p_i-configurations for path-tests.

If none of the vertices of a not properly placed p_i-branch (endpoints of a path piece) is incident with a vertex of a change-around pair, then no path can exist (see Fig. 4*a*).

p_i-containing 2-connected, nonseparable subgraphs according to Fig. 4*b* and 4*c* (or other case 2-rearranged subgraphs) should be "path-end"-marked (as suggested in the figure). If more than two such path-ends are obtained, then no path can exist.

If more than two p_i-branches are completely tied at a single vertex, then no path can exist (Fig. 4*d*).

With this method the path-formation has been referred to a recognition of 2-connected, nonseparable subgraphs. For a human observer this recognition is no problem, and should the method for some reason be programmed for a computer, it may be observed that change-around vertices and the 2-connected, nonseparable subgraphs may be systematically obtained at each step of the subrearrangement theorem.

Let us now apply the subrearrangement theorem and the path-formation method to a couple of examples.

Example 1. We want to investigate whether

$$B = ab \cup dfg \cup ac'deg \cup cefg \cup bc'ef$$

has an irredundant network. B contains no redundant literals, for it contains only prime implicants (compare (10)). The corresponding L_T-matrix is:

$$L_T = \begin{array}{|ccccccccc|} T & a & b & c & c' & d & e & f & g \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \end{array}.$$

Next L_T is transformed with row-replacements into an explicitly independent Γ_L-form:

$$\Gamma_L = \begin{array}{|ccccccccc|l} T & a & b & c & c' & d & e & f & g & \\ \hline 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 4 \end{array}$$

Γ_L generates the co-set c:

$$c = \begin{array}{|ccccccccc|} T & a & b & c & c' & d & e & f & g \\ \hline 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{array}$$

and hence we obtain for $B(c)$:

$$B(c) = B \cup abcde \cup bcc'df \cup acc'g.$$

The c-criterion is fulfilled, for the two last intersections vanish, and the other indicated clause of $B(c)$ subsumes the clause ab of B.

Let us now apply the subrearrangement theorem to Γ_L, for instance in the loop order 4, 3, 2, 1. The four steps are indicated in Fig. 5.

First loop 4 of Γ_L is drawn (Fig. 5*a*). The p_i-branches for loop 3, *e* and *d*, are already in a single path, and the corresponding graph is drawn (Fig. 5*b*). In order to form a path of the p_i-branches *c'* and *e* for loop 2 the series-connected branches *g* and *c'* are interchanged. The graph also containing loop 2 is drawn (Fig. 5*c*) and the p_i-branches for the final loop 1 are indicated. We have here three p_i-branches and two of them, for instance *d* and *g* are contained in a 2-connected, nonseparable subgraph (indicated in Fig. 5c). Since no path of p_i-branches outside the subgraph between its connection vertices exists, the two branches *d* and *g* must be rearranged into a path in the subgraph with an endpoint at one of its connection vertices. So the series-connected branches

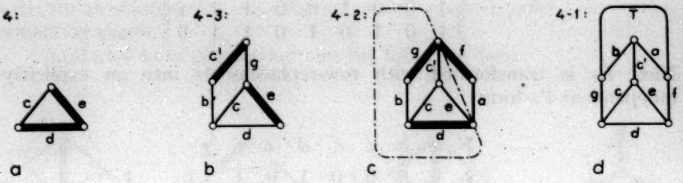


FIG. 5. G_i-constructions for Example 1.

b and *g* are interchanged. After that the subgraph is turned around and the desired path is formed. The remaining loop can now be adjoined and the irredundant graph of B is shown in Fig. 5*d*.

Example 2. In (2, p. 4–56), it is claimed that none of the following loop-sets

$$\left\|\begin{array}{c|cccc} \cdot & 0 & 1 & 1 & 1 \\ \cdot & 1 & 0 & 0 & 1 \\ \cdot & 1 & 0 & 1 & 0 \\ \cdot & 1 & 1 & 0 & 0 \end{array}\right\|, \quad \left\|\begin{array}{c|cccc} \cdot & 1 & 1 & 1 & 1 \\ \cdot & 1 & 0 & 0 & 1 \\ \cdot & 1 & 0 & 1 & 0 \\ \cdot & 1 & 1 & 0 & 0 \end{array}\right\|, \quad \left\|\begin{array}{c|cccc} \cdot & 1 & 1 & 1 & 1 \\ \cdot & 1 & 1 & 0 & 1 \\ \cdot & 1 & 0 & 1 & 0 \\ \cdot & 1 & 1 & 0 & 0 \end{array}\right\|,$$

has a loop-graph with the same number of branches (the four explicitly independent columns are deleted for shortness). In applying the subrearrangement theorem we will find that this is true for the two first, but not for the third Γ_L-set.

For the first:

$$\Gamma_L = \begin{array}{c} \begin{array}{cccc|cccc} a & b & c & d & e & f & g & h \end{array} \\ \left\|\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{array}\right\| \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

we obtain for the three first loops the graph of Fig. 6*a*, where the p_i-branches for loop 4 are indicated. No change-around vertices are incident with the vertices of e, and hence no graph for Γ_L.

For the second:

$$\Gamma_L = \begin{array}{c|cccc|cccc|c} & a & b & c & d & e & f & g & h & \\ \hline & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 2 \\ & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 3 \\ & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 4 \end{array}$$

we obtain for the three first loops the graph of Fig. 6*b*. The two p_i-branches for loop 4, e and f cannot be rearranged into a path, for e is not incident with a change-around vertex. Hence no graph for Γ_L.

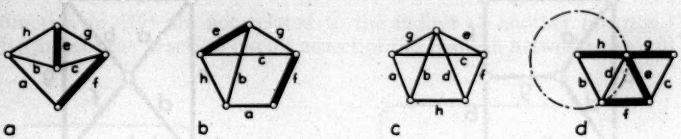


FIG. 6. Graphs of Example 2.

For the third:

$$\Gamma_L = \begin{array}{c|cccc|cccc|c} & a & b & c & d & e & f & g & h & \\ \hline & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 2 \\ & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 3 \\ & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 4 \end{array}$$

however, we immediately obtain a unique H^3_T-graph according to Fig. 6*c*. If we had applied the theorem in the loop order 4–3–2–1 we would for the loops 4–3–2 obtain the graph of Fig. 7*d*. This illustrates, what we have said in the above, that if in a 2-connected, nonseparable subgraph there is a p_i-path between its connection vertices outside the subgraph and also other p_i-branches in the outside (Case 2), then the p_i-branches inside the subgraph must form a single path only connected to one of the connection vertices. It is so here, and a change around of the subgraph gives a p_i-path.

Example 3. This example will illustrate that it is in general necessary that Γ_L is first transformed into an explicitly independent form

before the subrearrangement theorem can be applied. Let us consider the following two loop-generator sets of the same group G_L:

$$\Gamma_L = \left\| \begin{array}{cccccccc} a & b & c & d & e & f & g & h \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{array} \right\| \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \qquad \Gamma_L{}^* = \left\| \begin{array}{cccccccc} a & b & c & d & e & f & g & h \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{array} \right\|.$$

We want to see what can happen when the theorem is (incorrectly) applied to Γ_L, which is not in an explicitly independent form. After the three first steps (the three first loops of Γ_L), we obtain the G_i-graph of Fig. 7*a*. The process cannot be carried out further because the

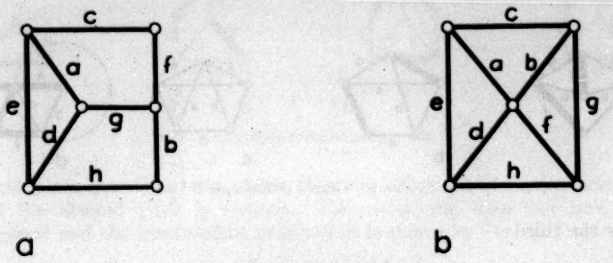


FIG. 7. Graphs of Example 3.

fourth loop does not contain any new branches. Although all the loops of G_i are correct (compare the complete graph of Fig. 7*b* which is obtained when the theorem is correctly applied on $\Gamma_L{}^*$), the structure of G_i has nothing to do with the structure of the complete graph.

We want to point out also, that it can happen that there is a new branch at each step in the application of the subrearrangement theorem on a Γ_L-set which is not explicitly independent. Anyhow the theorem should not be applied until Γ_L has been transformed in the prescribed form, for it can otherwise happen that a row of Γ_L represents a multiple loop.

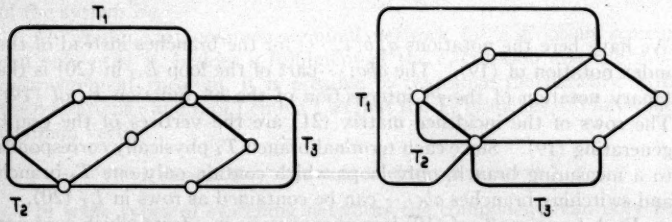
V. IRREDUNDANT n-TERMINAL BOOLEAN NETWORKS

We will here briefly outline that the c-criterion can be easily extended to cover the n-terminal case, and that the subrearrangement theorem is directly applicable also to n-terminal nets.

Suppose that we want to generate the system of Boolean functions B_k:

$$\left.\begin{aligned} B_1 &= \bigcup_j \Big(\bigcap_i^j a_{ij1}\Big) \\ B_2 &= \bigcup_j \Big(\bigcap_i^j a_{ij2}\Big) \\ &\vdots \\ B_m &= \bigcup_j \Big(\bigcap_i^j a_{ijm}\Big) \end{aligned}\right\} \tag{19}$$

The variables a_{ijk} are those literals which appear in the j^{th} intersection $\bigcap_i^j$ of the union $\bigcup_j$ of the k^{th} Boolean function. (The indices i, j of one function in (19) are not related to the indices of another function.) Because of the practical circuit aspect of the Boolean networks, we will



a. $T_k(3)$-graph. *b.* Common terminal $T_k(3)$-graph.

FIG. 8.

in the following distinguish between a $T_k(m)$-graph and a common-terminal $T_k(m)$-graph. Both types of graphs generate m functions which are measured in the m terminal branches T_k. In Fig. 8*a* we have drawn a $T_k(3)$-graph and in Fig. 8*b* a common-terminal $T_k(3)$-graph. For the graph of Fig. 8*a* we have no restrictions on the terminal branches T_k, but in a common-terminal graph (Fig. 8*b*) we require that all the m (= 3) T_k-branches shall have a common terminal vertex. Both types of graphs are assumed to be connected and nonseparable. Should some of the B_k-functions of (19) only contain literals, none of which is present in any of the remaining B_k-functions, the system will be divided into two separate systems and so on.

Evidently Eq. 1 is also valid for n-terminal graphs if the loop-matrix L_T and the incidence matrix H_{T^2} are supplied with one column for each T_k-branch:

$$L_T = \begin{array}{c|cccccccc} & T_1 & T_2 & T_3 \cdots & T_m & a & b & c \cdots \\ L_{11} & 1 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\ L_{12} & 1 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\ \vdots & & & & & & & \\ L_{21} & 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot \\ L_{22} & 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot \\ \vdots & & & & & & & \\ L_{m1} & 0 & 0 & 0 & 1 & \cdot & \cdot & \cdot \\ L_{m2} & 0 & 0 & 0 & 1 & \cdot & \cdot & \cdot \\ \vdots & & & & & & & \end{array} \tag{20}$$

$$H_{T^2} = \begin{array}{c|cccccccc} & T_1 & T_2 & T_3 \cdots & T_m & a & b & c \cdots \\ & & & & & & & \end{array} \tag{21}$$

We have here the notations a, b, c, $\cdots$ for the branches instead of the index notation of (19). The $abc\cdots$-part of the loop L_{kj} in (20) is the binary notation of the j^{th} intersection of the k^{th} function B_k of (19). The rows of the incidence matrix (21) are the vertices of the graph generating (19). Since each terminal branch T_k physically corresponds to a measuring branch, only loops which contain only one T_k-branch and switching branches $abc\cdots$ can be contained as rows in L_T (20).

Each function B_k of (19) is supposed to be in a form which contains no redundant literals.

We denote as before with Γ_H and Γ_L generator sets to the groups G_H and G_L.

Let us divide Γ_L into:

$$\Gamma_L = \gamma_1 \cup \gamma_2 \cup \cdots \cup \gamma_m \cup \Gamma \tag{22}$$

where γ_k is an element of Γ_L with a 1 in the T_k-position and 0's in the other T_i-positions. The generator set Γ contains 0's in all T_i-positions. Γ generates the subgroup g. Expanding G_L in g and co-sets, we are only interested in the co-sets c_k with the co-set-leaders γ_k:

$$c_k = \gamma_k + g \bmod 2. \tag{23}$$

The elements of c_k are the only elements of G_L which contain a 1 in the T_k-position and 0's in the other T_r-positions. A co-set c_k is uniquely derived from all the B_k-functions. From c_k we can form the Boolean function $B(c_k)$ and we must require that the following criterion is fulfilled:

c_k-criterion: A necessary condition for a system of Boolean forms B_k (19) to have an irredundant network is that the function $B(c_k)$, which corresponds to the co-set c_k (23) of L_T, implies B_k:

$$B(c_k) \subseteq B_k$$
$$k = 1, 2, \cdots, m. \tag{24}$$

We say as for two-terminal networks that a Boolean system B_k ($k = 1, 2, \cdots, m$) is generated by (corresponds to) an irredundant network, if the network generates the form $B(c_k)$ which implies B_k.

Evidently the subrearrangement theorem is valid also in the n-terminal case. It solves together with the c_k-criterion the existence problem for irredundant $T_k(m)$-networks as referred to a specific form of the system B_k.

An irredundant common-terminal network for a Boolean system can only exist if, besides a fulfilment of the c_k-criterion and the subrearrangement theorem, it is possible to rearrange the network with the operations *i* and *ii* of Lemma 1 so that the T_k branches will have one common vertex.

VI. EXTENSION TO NETWORKS OF ORIENTED BRANCHES

In some types of switching networks the components can only be conducting in one direction. Their state, conducting or nonconducting, may as in the case of nonoriented branches be controlled by input quantities, one for each component.

In these oriented networks (compare (**15, 3, 1, 4**)) a loop is represented with a matrix row containing the numbers + 1, − 1, and 0. If a branch is incident with the loop and its direction is in one sense relative to the loop, there is a + 1 in the corresponding branch position. If the branch has the opposite direction, there is a − 1. If the branch is not incident with the loop, there is a 0.

In the branch-vertex incidence matrix, the number + 1 indicates the plus-vertex of a branch and the number − 1, the minus-vertex of a branch. A zero indicates that a branch is not incident with a vertex.

With this extension, Eq. 1 still holds. The matrix product (row-column rule) is however defined with ordinary multiplication and addition (not mod 2 addition).

To a Boolean form we can associate a loop-set matrix L_T. In trying to give L_T an explicitly independent form with the row-replacement procedure (one row is added to, or subtracted from another row) it can happen that the Γ_L-set will contain rows with + 1's, − 1's, and 0's. Such a row will be called a 1-element row of the matrix. A row containing other digits (for instance + 2) can eventually be interpreted as a multiple loop, but since we are only interested in single loops, we want to transform L_T so as to contain only 1-element rows.

It is immediately clear that if a graph exists, it must always be possible to generate a 1-element row from two 1-element rows. (Matrices with this property have been studied in connection with circuit theory by Cederbaum (1).) An immediate consequence is that if a graph exists, then it is always possible to transform L_T into an explicitly independent generator set Γ_L. All 1-element rows with a + 1 in the T-element which are generated by Γ_L must be in agreement with the prescribed Boolean function.

The final condition on Γ_L in order that an irredundant network shall exist (or that a graph shall exist) is a slight modification of the subrearrangement theorem.

Because of the directed branches we are not allowed to make a single turn-around operation on a 2-connected subgraph. Only pure series-interchanges of branch orders in a loop which maintain the direction of the branches are allowed. So the subrearrangement theorem should be modified so that at each step it must be possible to arrange intended path-branches p_i into a single path with the prescribed branch-orientation. And such an arrangement must only be made with a direction-maintaining series-interchange of the branches in a loop. This makes the path-formation method even easier.

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A GRAVITATIONAL FORCE FUNCTION FOR THE EARTH REPRESENTING ALL DEVIATIONS FROM A SPHERICAL GEOID

BY

RAYMOND H. WILSON, JR.[1]

ABSTRACT

In the present treatment each area of deviation from a uniform spherical gravitational field is represented by a separate series of Legendre polynomials having its pole of reference centered on the area. Thus the Earth's north and south polar oblateness deviations are represented separately so that their inequality can be investigated. The series for minor deviations centered at general latitudes and longitudes has been transformed to terms of variables in the inertial coordinate system for practical computation. The vector sum of these expressions for the Earth's spherical gravitational field and its deviations, together with explicitly presented geocentric expressions for the fields of the moon and sun, would thus constitute a complete force function for satellite orbit development by numerical integration.

It was essentially proved by Newton (1)[2] that a solid sphere whose density is a function only of the distance from its center attracts any external particle as though the whole mass were at the center. Thus, using proper canonical units for mass, length, and time, the spherical potential,

$$V = \frac{1}{r} \tag{1}$$

where r is the distance from the center. Then the gravitational force along any line is always the partial derivative of V along that line, so that, for this V, the radial force[3]

$$F = -\frac{1}{r^2}. \tag{2}$$

[1] National Aeronautics and Space Administration, Washington, D. C.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] Using general units in the force function, Newton's Second Law of Motion states that the resulting radial acceleration of an external particle has the dimensional equation:

$$\frac{k^2 M}{r^2} = \frac{\text{linear units}}{(\text{time units})^2} = g,$$

where k^2 is the gravitational constant for the attracting spherical mass M, and its value depends on the units chosen. Units producing $k^2 M = 1$ are said to be "canonical," and any two of them may be chosen arbitrarily, provided that the third unit is adjusted accordingly. If, for the Earth, we choose its mass M_E and equatorial radius r_E to be unity, then k_E^2 will also be unity, provided that the acceleration of gravity g_0' at the equator (of a non-rotating spherical Earth) is unity when expressed in the canonical units. This condition is expressed by the

(continued on p. 379)

It can be shown (2) that, when a deviation from geoidal sphericity is symmetric about the half-line axis from which the central angle θ is measured, as in Fig. 1, representation of the corresponding correction

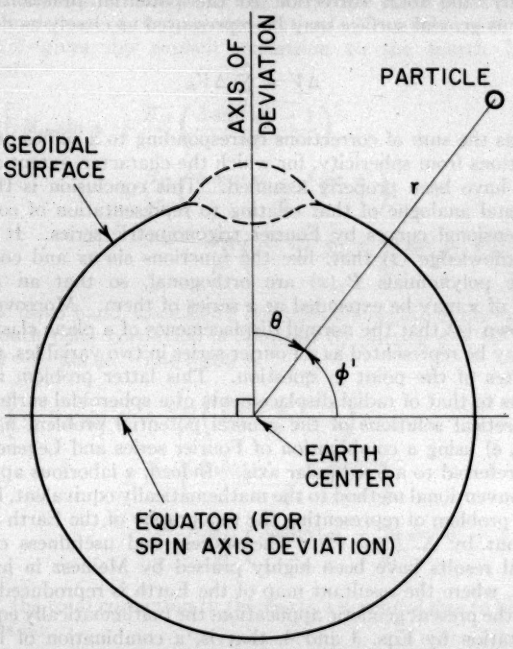

FIG. 1. Section through the center of the Earth, the attracted particle, and the point of deviation from geoidal sphericity (deviation shown much exaggerated).

to the potential function for the m^{th} deviation is given by the infinite series:

$$\Delta V_m = \sum_{n=1}^{\infty} \frac{B_{mn} P_n(\cos\theta)}{r^{n+1}} \tag{3}$$

equation:

$$g_0' = 1 = \left(\frac{g_0}{r_E}\right) \Big/ \left(\frac{1}{t_0}\right)^2,$$

where t_0 is the canonical time unit. A solution for t_0, using accepted measured values $r_E = 6{,}378{,}388$ meters and $g_0 = 9.81426$ meters per sec.^{2} gives $t_0 = \sqrt{r_E/g_0} = 806.170$ sec., in which the sixth digit, like the quantities from which it is derived, is uncertain; improvement should result as feedback from comparison of theory with further observations. Also, all force function terms added to represent deviations from a sphere must be included to obtain a correct working time unit.

where $P_n(\cos\theta)$ is the Legendre polynomial or surface zonal harmonic of n^{th} order, and the coefficients B_{mn} are arbitrary constants depending on the character and extent of the deviation.

Finally, the total correction to the potential function for any continuous geoidal surface may be represented as closely as desired by

$$\Delta V = \sum_{1}^{m} \Delta V_m \tag{4}$$

that is, as the sum of corrections corresponding to a sufficient number of deviations from sphericity, for which the character, extent and axial location have been properly assumed. This conclusion is the three-dimensional analogue of that relating to representation of continuous two-dimensional curves by Fourier trigonometric series. It is based on the knowledge (3) that, like the functions $\sin nx$ and $\cos nx$, the Legendre polynomials $P_n(x)$ are orthogonal, so that an arbitrary function of x may be expanded as a series of them. Moreover, it has been shown (4) that the normal displacements of a plane elastic membrane may be represented as a Fourier series in two variables, the plane coordinates of the point in question. This latter problem is clearly analogous to that of radial displacements of a spheroidal surface.

Theoretical solutions of the general potential problem have been given (5, 6) using a combination of Fourier series and Legendre polynomials referred to a fixed polar axis. Indeed, a laborious application of this conventional method to the mathematically equivalent, but more difficult, problem of representing the topography of the Earth has been carried out by A. Prey (7). The success and usefulness of Prey's numerical results have been highly praised by Meinesz in his recent book (8), where the resultant map of the Earth is reproduced. However, for the present geodetic application, the mathematically equivalent representation by Eqs. 3 and 4, that is, a combination of Legendre polynomials having different polar axes, would seem more feasible.

Practical determination of the corrections for a particular spheroidal surface would be most convenient when two or three of the largest ΔV's are specified first; then further ones of smaller order worked out as corrections to those considered in the previous step. The remainder of the present paper is devoted to application of the above principles to obtain gravitational potential and force functions for the Earth.

The most outstanding deviations from gravitational uniformity in the case of the Earth are the two maxima which appear approximately to be centered at the north and south poles of rotation. Only if the Earth were homogeneous and in isostatic equilibrium, could one expect the centering to be exact, and the maxima to be uniformly equal. On the present approach, neither assumption need be made permanently, although, for the first approximation, the conveniently determinate

spin axis of the Earth will be taken as the axis of both the north and south polar maximum deviations, especially since existing gravimetric data are probably insufficient to improve on that as a tentative assumption, anyway. Then, as in Fig. 1, for the north polar deviation, θ is the complement of the geocentric latitude ϕ', so that $\cos\theta = \sin\phi'$, and Eq. 3 gives, by explicit expansion to the fourth Legendre polynomial:

$$\Delta V_1 = \frac{1}{r^2}\left[B_{11}\sin\phi' + \frac{B_{12}}{r}\left(\frac{3\sin^2\phi' - 1}{2}\right) + \frac{B_{13}}{r^2}\left(\frac{5\sin^3\phi' - 3\sin\phi'}{2}\right) + \frac{B_{14}}{r^3}\left(\frac{35\sin^4\phi' - 30\sin^2\phi' + 3}{8}\right)\right]. \quad (5)$$

On the other hand, since south latitudes are negative, the expression for the south polar correction is obtained by putting $\theta = 90° + \phi'$ so that $\cos\theta = -\sin\phi'$. The result is

$$\Delta V_2 = \frac{1}{r^2}\left[-B_{21}\sin\phi' + \frac{B_{22}}{r}\left(\frac{3\sin^2\phi' - 1}{2}\right) - \frac{B_{23}}{r^2}\left(\frac{5\sin^3\phi' - 3\sin\phi'}{2}\right) + \frac{B_{24}}{r^3}\left(\frac{35\sin^4\phi' - 30\sin^2\phi' + 3}{8}\right)\right]. \quad (6)$$

Thus, the total correction to the Earth's potential due to both polar deviations is:

$$(\Delta V_1 + \Delta V_2) = \frac{1}{r^2}\left[(B_{11} - B_{21})\sin\phi' + \frac{(B_{12} + B_{22})}{r}\left(\frac{3\sin^2\phi' - 1}{2}\right) + \frac{(B_{13} - B_{23})}{r^2}\left(\frac{5\sin^3\phi' - 3\sin\phi'}{2}\right) + \frac{(B_{14} + B_{24})}{r^3}\left(\frac{35\sin^4\phi' - 30\sin^2\phi' + 3}{8}\right)\right]. \quad (7)$$

Here the coefficients of the odd terms, being differences between those of corresponding north and south terms, can be expected to be relatively small. Indeed, only the even terms, giving corrections for a true ellipsoid, have been regularly included by other authors in representing the Earth's potential for geodetic and astronomical purposes. Ac-

cepted coefficient values (9) in canonical units are $(B_{12} + B_{22}) = -0.0010917$ and $(B_{14} + B_{24}) = +0.0000024$. However, more refined gravimetric data from the polar regions, as well as close Earth satellite orbital perturbations, may soon make the odd-term coefficients detectible (10).

Satellite orbit developments by numerical integration result most conveniently from expressions for three force components in the

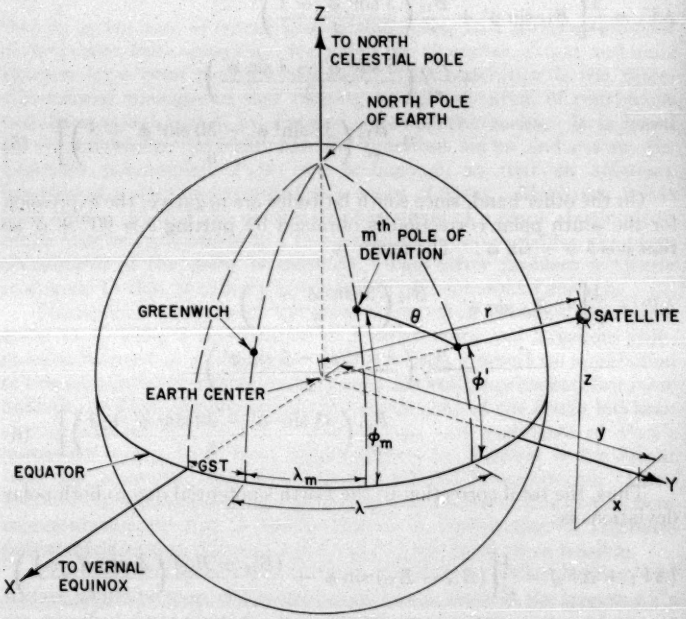

FIG. 2. Geometrical relations of the inertial and geographical coordinate systems as to positions of a satellite and any pole (center) of deviation of the geoidal potential function from exact spherical form.

positive inertial directions defined by: (x) the vernal equinox, (y) the projection of the summer solstice on the celestial equator, and (z) the north celestial pole, as shown in Fig. 2. Substituting in Eq. 7: $\sin \phi' = z/r$ and $r = \sqrt{x^2 + y^2 + z^2}$, and differentiating with respect to x, y, and z, one obtains, respectively, as total correction to the force components:

$$
\begin{aligned}
(\Delta G_1 + \Delta G_2)_x &= -\frac{x}{r^3}\Big[3(B_{11} - B_{21})\frac{z}{r^2} + \frac{3(B_{12} + B_{22})}{2r^4}(5z^2 - r^2) \\
&\quad + \frac{5z(B_{13} - B_{23})}{2r^6}(7z^2 - 3r^2) \\
&\quad + \frac{5(B_{14} + B_{24})}{8r^8}(63z^4 - 42r^2z^2 + 3r^4)\Big] \\
(\Delta G_1 + \Delta G_2)_y &= -\frac{y}{r^3}\Big[3(B_{11} - B_{21})\frac{z}{r^2} + \frac{3(B_{12} + B_{22})}{2r^4}(5z^2 - r^2) \\
&\quad + \frac{5z(B_{13} - B_{23})}{2r^6}(7z^2 - 3r^2) \\
&\quad + \frac{5(B_{14} + B_{24})}{8r^8}(63z^4 - 42r^2z^2 + 3r^4)\Big] \qquad (8) \\
(\Delta G_1 + \Delta G_2)_z &= -\frac{z}{r^3}\Big[\frac{(B_{11} - B_{21})}{r^2}\frac{(3z^2 - r^2)}{z} \\
&\quad + \frac{3(B_{12} + B_{22})}{2r^4}(5z^2 - 3r^2) \\
&\quad + \frac{(B_{13} - B_{23})}{2r^6}\frac{(35z^4 - 30z^2r^2 + 3r^4)}{z} \\
&\quad + \frac{5(B_{14} + B_{24})}{8r^8}(63z^4 - 70r^2z^2 + 15r^4)\Big].
\end{aligned}
$$

The corrections given by (8) are to be added to the force components given by (2) for a perfect sphere; namely:

$$
\begin{aligned}
G_x &= -\frac{x}{r^3} \\
G_y &= -\frac{y}{r^3} \qquad (9) \\
G_z &= -\frac{z}{r^3}
\end{aligned}
$$

to give the Earth's gravitational force components including polar flattening.

Synoptic charts of gravimetric data also show areas of lesser deviation, positive and negative, each roughly symmetric about the geographical points of their respective maximum or minimum. Taking these points as locations of axes of symmetry, the corresponding corrections to the potential result by substituting in (3):

$$\cos\theta = \sin\phi' \sin\phi_m' + \cos\phi' \cos\phi_m' \cos(\lambda - \lambda_m) \tag{10}$$

where λ_m is the longitude and ϕ_m' the geocentric latitude of the m^{th} pole of deviation. The longitude λ of the point at which the potential is being represented may be transferred, for any instant of time, to the XYZ inertial coordinate system by the transformations:

$$\sin\lambda = \frac{y}{r}\cos GST - \frac{x}{r}\sin GST$$

$$\cos\lambda = \frac{x}{r}\cos GST + \frac{y}{r}\sin GST \tag{11}$$

where GST is the Greenwich Sidereal Time at the instant in question and the geometrical situation is illustrated in Fig. 2. Hence, for the m^{th} deviation, the potential correction by (3) is:

$$\Delta V_m = \frac{B_{m1}}{r^2}\left\{\sin\phi' \sin\phi_m' + \cos\phi' \cos\phi_m' \left[\cos\lambda_m \left(\frac{x}{r}\cos GST + \frac{y}{r}\sin GST\right) + \sin\lambda_m \left(\frac{y}{r}\cos GST - \frac{x}{r}\sin GST\right)\right]\right\}. \tag{12}$$

Only one term is here detailed for this smallest order deviation having a general geographical center, as being practically sufficient for representing any prospectively available data. A general expression for (12) may be written by means of the addition theorem (11), although it is of less practical interest here. It is

$$\Delta V_m = \sum_{n=1}^{\infty} \sum_{l=-n}^{+n} \frac{B_{mn}}{r^{n+1}} \left(\frac{2}{2n+1}\right) \overline{P_n^l}(\sin\phi') \overline{P_n^l}(\sin\phi_m') e^{il(\lambda - \lambda_m)}. \tag{13}$$

Here $\overline{P_n^l}$ denotes a normalized associated Legendre function of the first kind, corresponding to the l^{th} derivative of an n^{th} order Legendre polynomial; and $i = \sqrt{-1}$. It is to be noted that Eqs. 5 and 6 are merely special cases of (13), representing deviations assumed to be centered on the geographical poles of rotation.

The coefficient of the first corrective term should have the same sign as that of the gravimetric deviation. Its order of size may be numerically about that of the amount of maximum deviation in kilogalileos, when the deviation is measured from the ellipsoid represented by the even-order terms of (7). For instance, for a maximum deviation, on the usual synoptic map, of + 20 milligals, + 0.00002 would be the order of magnitude of $(B_{11} - B_{21})$ in (7), or of B_{m1} in (12).

By substituting in (12) $\sin\phi' = z/r$, $\cos\phi' = \sqrt{x^2 + y^2}/r$ and $r = \sqrt{x^2 + y^2 + z^2}$, and differentiating with respect to x, y, and z, one obtains as corresponding corrections to the three force components:

$$\Delta G_{mx} = -B_{m1}\frac{x}{r^3}\left\{\frac{3z}{r^2}\sin\phi_m' + \cos\phi_m'\left[(\cos\lambda_m\cos GST - \sin\lambda_m\sin GST)\frac{(4x^4+4x^2y^2-2x^2r^2-r^2y^2)}{xr^3\sqrt{x^2+y^2}} + \frac{(\cos\lambda_m\sin GST+\sin\lambda_m\cos GST)(4x^2y+4y^3-r^2y)}{r^3\sqrt{x^2+y^2}}\right]\right\}$$

$$\Delta G_{my} = -B_{m1}\frac{y}{r^3}\left\{\frac{3z}{r^2}\sin\phi_m' + \cos\phi_m'\left[(\cos\lambda_m\cos GST - \sin\lambda_m\sin GST)\frac{(4x^3+4xy^2-r^2x)}{r^3\sqrt{x^2+y^2}} + \frac{(\cos\lambda_m\sin GST+\sin\lambda_m\cos GST)(4x^2y^2+4y^4-2y^2r^2-r^2x^2)}{yr^3\sqrt{x^2+y^2}}\right]\right\} \tag{14}$$

$$\Delta G_{mz} = -B_{m1}\frac{z}{r^3}\left\{\frac{3z^2-r^2}{zr^2}\sin\phi_m' + \cos\phi_m'\left[(\cos\lambda_m\cos GST - \sin\lambda_m\sin GST)\frac{4x\sqrt{x^2+y^2}}{r^3} + (\cos\lambda_m\sin GST+\sin\lambda_m\cos GST)\frac{4y\sqrt{x^2+y^2}}{r^3}\right]\right\}.$$

Equations 14 for a sufficient number of minor geoidal deviations should, when added to (8) and (9), represent the Earth's gravitational force vector $\mathbf{G}_E$ with appropriate exactitude. However, even for close satellites, the external gravitational fields of the moon $\mathbf{G}_M$, and of the sun $\mathbf{G}_S$, approach the minor geoidal deviations in order of importance. Hence explicit geocentric expressions for these extra-terrestrial forces, adapted from those of Kooy (12), are given below without discussion, for completeness of satellite orbit development by numerical integration.

The components of $\mathbf{G}_M$ are:

$$G_{Mx} = -\frac{1}{81.45}\left[\frac{(x-x_M)}{[(x-x_M)^2+(y-y_M)^2+(z-z_M)^2]^{3/2}} + \frac{x_M}{(x_M^2+y_M^2+z_M^2)^{3/2}}\right]$$

$$G_{My} = -\frac{1}{81.45}\left[\frac{(y-y_M)}{[(x-x_M)^2+(y-y_M)^2+(z-z_M)^2]^{3/2}} + \frac{y_M}{(x_M^2+y_M^2+z_M^2)^{3/2}}\right] \tag{15}$$

$$G_{M_z} = -\frac{1}{81.45}\left[\frac{(z - z_M)}{[(x - x_M)^2 + (y - y_M)^2 + (z - z_M)^2]^{3/2}} + \frac{z_M}{(x_M^2 + y_M^2 + z_M^2)^{3/2}}\right]$$

where

$$x_M = \frac{\cos\delta_M \cos\alpha_M}{\sin\pi_M}$$
$$y_M = \frac{\cos\delta_M \sin\alpha_M}{\sin\pi_M} \qquad (16)$$
$$z_M = \frac{\sin\delta_M}{\sin\pi_M}.$$

In (16), α_M, δ_M, and π_M are the right ascension, declination, and parallax, respectively, of the moon, which are tabulated in the *American Ephemeris*.

The components of $\mathbf{G}_S$ are:

$$G_{S_x} = -333{,}432\left\{\frac{(x - x_S)}{[(x - x_S)^2 + (y - y_S)^2 + (z - z_S)^2]^{3/2}} + \frac{x_S}{(x_S^2 + y_S^2 + z_S^2)^{3/2}}\right\}$$
$$G_{S_y} = -333{,}432\left\{\frac{(y - y_S)}{[(x - x_S)^2 + (y - y_S)^2 + (z - z_S)^2]^{3/2}} + \frac{y_S}{(x_S^2 + y_S^2 + z_S^2)^{3/2}}\right\} \qquad (17)$$
$$G_{S_z} = -333{,}432\left\{\frac{(z - z_S)}{[(x - x_S)^2 + (y - y_S)^2 + (z - z_S)^2]^{3/2}} + \frac{z_S}{(x_S^2 + y_S^2 + z_S^2)^{3/2}}\right\}$$

In (17):

$$x_S = 23439\,X$$
$$y_S = 23439\,Y \qquad (18)$$
$$z_S = 23439\,Z$$

where X, Y, Z, are the solar coordinates tabulated daily in the *American Ephemeris*.

An additional force due to the sun, which would have a noticeable effect on very small or low-density (for example, "balloon") satellites,

is its radiation pressure. This may conveniently be included by adding $+ 25.4 \left(\frac{A}{m}\right) \left\{ \frac{(x - x_S)}{[(x - x_S)^2 + (y - y_S)^2 + (z - z_S)^2]^{3/2}} \right\}$ to G_{Sx}, and analogous terms to the y and z force components in (17). Here A is the profile area in cm² normal to the solar radius vector, and m is the mass in grams, of the satellite; for other dimensions canonical units, such as the Earth's equatorial radius, are to be used. Of course, these radiation pressure terms must be omitted during intervals when the satellite is shaded by the Earth. A method for determining such eclipse intervals has been outlined by the present author (13).

Similarly, radiation pressure due to reflection and re-radiation of sunlight from the Earth could be taken into account by adding the mean amount $+ 1.16 \times 10^{-8} \left(\frac{A}{m}\right) \frac{1}{r^2}$ to Eq. 2, with the constant varying about $\pm$ 30% from "full" to "new" Earth. Such terrestrial radiation has been discussed by the author in *Science* **127**, 812, and **128**, 209 (1958). All radiation pressure coefficients here assume a black isothermal body.

Thus, the vector sum:

$$\mathbf{G} = \mathbf{G}_E + \mathbf{G}_M + \mathbf{G}_S \tag{19}$$

given by Eqs. 8, 9, 14, 15, and 17, represents the complete geocentric gravitational force in the differential equations of close Earth satellite motions.

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UNIVERSAL UNITS OF MAGNETISM, MECHANICS AND TEMPERATURE

BY

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ABSTRACT

The ever-increasing accuracy in measurements of fundamental quantities may eventually lead to the establishment of five highly precise "universal units" of magnetic intensity, mass, length, time and temperature. Speculations on the current possibilities for the selection and use of such units are presented. It is proposed that these hypothetical units may permit the revelation of many new universal constants of major importance to fundamental theory and to causal resolutions of several stubborn mysteries of physics.

Suggested practical values for new units are: magnetic intensity unit $M = 1.84253$ gauss; mass unit $m = 217.699$ grams; length unit $r = 1.61562$ centimeters; time unit $t = 0.538912$ sec.; temperature unit $^{\circ}T = 0.0708681$ degree Absolute.

INTRODUCTION

The 1958 edition of "The Handbook of Physics" (1)[2] records the latest values for the fundamental constants of physics and surveys yearly progress in the accuracy of their measurement. The event of each new publication of such data serves as a reminder of interesting historic speculations (2, 3, 4, 5) on the subject of universal physical units of mass, space, time and temperature. For instance, in 1957, P. A. M. Dirac (2) tentatively proposed a unit of mass equal to $\hbar m_e c e^{-2}$, a unit of length equal to $e^2(m_e c^2)^{-1}$, and a unit of time equal to $e^2(m_e c^3)^{-1}$. Similarly, Sir Arthur S. Eddington (3) once suggested mass, space, time and temperature units equal to $(\hbar c G^{-1})^{1/2}$, $(Ghc^{-3})^{1/2}$, $(Ghc^{-5})^{1/2}$, and $(hc^6/Gk^2)^{1/2}$, respectively. H. Bondi (5) and E. J. Zimmerman (6) have summarized these and other speculations on the subject by various scientists and philosophers. They also describe the evidence of a cosmological constant, a pure number equal approximately to 10^{40}, and its probable importance in eventual unification of the laws and constants of classical mechanics and relativity with those of quantum mechanics, electromagnetism, and atomic theory.

This study suggests, on the basis of evidence presented previously (7, 8), that it is now possible to select an additional universal unit of magnetism, that is, $c^3\alpha_e(Gm_e)^{-1}$ or $2\ m_e M_B c^4(G\hbar^2)^{-1}$. It is believed that such a selection can be vitally important to resolution of several mysteries which have complicated fundamental theory and contributed heavily to the present "crisis in physics."

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

UNIVERSAL PHYSICAL UNITS

The 1958 values (1) for the fundamental quantities may be employed, in a manner similar to that of Dirac (2) and Eddington (3), to define quite quantitative universal units, as follows:

$$m = 2.17699 \times 10^{-5} \text{ g.} = \text{unit of mass} \tag{1}$$

$$r = 1.61562 \times 10^{-33} \text{ cm.} = \text{unit of radius (or length)} \tag{2}$$

$$\omega = 1.85559 \times 10^{43} \text{ radians} \cdot \text{sec.}^{-1} = \text{unit of angular velocity} \tag{3}$$

$$t = 5.38912 \times 10^{-44} \text{ sec.} = \omega^{-1} = \text{unit of time}$$

$$T = 7.08681 \times 10^{31} \text{ °A} = \text{unit of temperature} \tag{4}$$

$$B = 1.84253 \times 10^{56} \text{ gauss} = \text{unit of magnetic intensity} \tag{5}$$

$$(B_e = 3.23640 \times 10^{63} \text{ sec.}^{-1} = 1.84253 \times 10^{56} \text{ gauss} = \text{alternate to } B)$$

Note that the latter value assumes the postulated equality: gauss unit $= 1.7589 \times 10^{7}$ sec.$^{-1}$, as previously suggested (7, 8). Though some of the above units have strange qualities, for example, the magnitudes of certain exponents, their use in the following derivations of well-known fundamental quantities will clarify their origins, for example, employing the first three units,

$$r\omega = \frac{r}{t} = 2.99793 \times 10^{10} \frac{\text{cm.}}{\text{sec.}} = c, \tag{6}$$

$$mr^2\omega = 1.05443 \times 10^{-27} \frac{\text{g. cm.}^2}{\text{sec.}} = \hbar = \frac{h}{2\pi}, \tag{7}$$

$$\frac{r^3\omega^2}{m} = 6.670 \times 10^{-8} \frac{\text{cm.}^3}{\text{g. sec.}^2} = G, \tag{8}$$

Note that quantities 6, 7 and 8 are the well-known universal constants of light velocity, "angular momentum" (or Planck's constant divided by 2π), and gravitation.

$$mr = 3.51719 \times 10^{-38} \text{ g. cm.} = \frac{\hbar}{c} = \frac{h}{2\pi c}, \tag{9}$$

$$\frac{m}{\omega} = 1.17321 \times 10^{-48} \text{ g. sec.} = \frac{\hbar}{c^2} = \frac{h}{2\pi c^2}, \tag{10}$$

$$mr^3\omega^2 = 3.16111 \times 10^{-17} \frac{\text{g. cm.}^3}{\text{sec.}^2} = \hbar c = \frac{hc}{2\pi}, \tag{11}$$

$$\frac{r}{m} = 7.421 \times 10^{-29} \frac{\text{cm.}}{\text{g.}} = \frac{G}{c^2}, \tag{12}$$

$$\frac{r^2\omega}{m} = 2.222 \times 10^{-18} \frac{\text{cm.}^2}{\text{g. sec.}} = \frac{G}{c}, \text{ and} \tag{13}$$

$$m\omega = 4.040 \times 10^{28} \frac{\text{g.}}{\text{sec.}} = \frac{c^3}{G}. \tag{14}$$

Each of these constants has played an important role in the development of either submicroscopic particle physics (quantum mechanics and atomic theory) or astronomic particle physics (classical mechanics, gravitation, relativity, and cosmological theories). The fourth unit, that is, of temperature, employed in conjunction with the first three, may be understood from derivations of the following constants of thermodynamic and radiation theories, that is

$$\frac{mr^2\omega^2}{2T} = 1.38044 \times 10^{-16} \frac{\text{g. cm.}^2}{\text{sec.}^2\ ^\circ A} = k \tag{15}$$

$$\frac{m\omega^3}{\pi^4 T^4} = 5.66105 \times 10^{-5} \frac{\text{g.}}{\text{sec.}^3\ ^\circ A^4} = \sigma \tag{16}$$

$$\frac{4m\omega^2}{r\pi^4 T^4} = 7.5532 \times 10^{-15} \frac{\text{g.}}{\text{cm. sec.}^2\ ^\circ A^4} = a \tag{17}$$

$$4\pi rT = 1.43880 \times 10^0 \text{ cm. } ^\circ A = \lambda\,^\circ A. \tag{18}$$

Note that quantities 15, 16, 17 and 18 include the familiar Boltzmann's constant, the second radiation constant, the Stefan-Boltzmann's constant, and the radiation density constant.

The fifth unit, employed in conjunction with the first three, permits the additional derivation of certain electronic and electromagnetic constants (or numerical coefficients which will be readily recognized as vital in the mathematics of electromagnetic theory) e.g.,

$$\frac{rB}{\omega} = 1.60206 \times 10^{-20} \text{ cm. sec.} \cdot \text{gauss}^*$$

$$= 2.81787 \times 10^{-13} \text{ cm.} = \frac{m_e M_B}{S_e} = \frac{\hbar\alpha_e}{m_e c} \tag{19}$$

$$r^2 B = 4.80286 \times 10^{-10} \text{ cm.}^2 \text{ gauss}^*$$

$$= 8.44776 \times 10^{-3} \frac{\text{cm.}^2}{\text{sec.}} = \frac{e^2}{\epsilon_0 m_e c} = \frac{\hbar\alpha_e}{m_e}. \tag{20}$$

* Note that each of these quantities may be converted to oersted units by the equality, $\text{cm.}^3 \text{ gauss} = \text{g. cm.}^2 \text{ sec.}^{-2} \text{ oersted}^{-1}$.

Note that the coefficients of quantity 19 are equal to those usually associated with the emu of "charge" and/or the "classical radius of the electron." The first coefficient of quantity 20 is equal to that of the esu of "charge."

If the electron's well-known rest mass, $m_e = 9.1083 \times 10^{-28}$ g., is combined with units 1, 2, 3, and 5, additional recognizable electronic constants (or their coefficients) can be derived, for example,

$$\frac{mr^3B}{2m_e} = 9.28370 \times 10^{-21} \text{ cm.}^3 \text{ gauss}^*$$

$$= 1.63291 \times 10^{-13} \frac{\text{cm.}^3}{\text{sec.}} = \frac{\hbar^2\alpha_e}{2m_e^2c} = M_B \quad (21)$$

$$\frac{rB}{m_e\omega} = 1.7589 \times 10^7 \frac{\text{cm. sec.} \cdot \text{gauss}^*}{\text{g.}}$$

$$= 3.09373 \times 10^{14} \frac{\text{cm.}}{\text{g.}} = \frac{\hbar\alpha_e}{m_e^2c} = \mu_e \quad (22)$$

$$\frac{B}{4\pi m_e\omega^2} = 4.66885 \times 10^{-5} \frac{\text{gauss}^* \text{ sec.}^2}{\text{g.}}$$

$$= 8.21204 \times 10^2 \frac{\text{sec.}}{\text{g.}} = \frac{\hbar\alpha_e}{4\pi m_e^2c^2} \quad (23)$$

$$\frac{2\pi rm}{m_e} = 2.42626 \times 10^{-10} \text{ cm., that is, Compton's wave-length.} \quad (24)$$

Note that 21 is the familiar Bohr magneton, the electron's magnetic moment. Also, 22 will be noted as being equivalent to the author's proposed (**7, 8**) value for the magnetic permeability constant of free-space. The first coefficient of 23 will be observed as equal to that of the Zeeman displacement constant. The dimensions may be transposed to $(\text{cm.} \cdot \text{oersted})^{-1}$.

Note also, in the fundamental constants which define 19 through 23, that α_e is the well-known electronic fine structure constant, and e is the specific electronic charge, that is, equal to $(\hbar c\alpha_e\epsilon_0^{-1})^{1/2}$, where $\epsilon_0 = 1.0$. Additional significance of the substitute unit B_e may then be observed in the following derivations:

$$\frac{m_e}{r^3\omega B} = 3.59645 \times 10^{-36} \frac{\text{g. sec.}^2}{\text{cm.}^3}$$

$$= 6.3258 \times 10^{-29} \frac{\text{g. sec.}}{\text{cm.}^3 \text{ gauss}} = \frac{m_e^2}{\hbar c\alpha_e} = \frac{m_e^2\epsilon_0}{e^2} \quad (25)$$

$$\frac{m_e B_a}{m\omega} = 7.29729 \times 10^{-3}$$

$$= 4.14878 \times 10^{-10}\ \text{gauss}\cdot\text{sec.} = \alpha_e = \frac{e^2}{\epsilon_0 \hbar c} \quad (26)$$

$$m_e r^2 \omega B_a = 23.067517 \times 10^{-20} \frac{\text{g. cm.}^3}{\text{sec.}^2}$$

$$= 1.31147 \times 10^{-26} \frac{\text{g. cm.}^3\ \text{gauss}}{\text{sec.}} = \frac{e^2}{\epsilon_0} = \hbar c \alpha_e \quad (27)$$

$$\frac{m^2 r \omega}{m_e^2 B_a} = 0.529172 \times 10^{-8}\ \text{cm.} = 9.30761 \times 10^{-2} \frac{\text{cm.}}{\text{sec.}\cdot\text{gauss}} \quad (28)$$

$$\frac{m_e^2 r^2 B_a^2}{2m^2} = 2.17958 \times 10^{-11} \frac{\text{g. cm.}^2}{\text{sec.}^2}$$

$$= 7.04518 \times 10^{-26}\ \text{g. cm.}^2\ \text{gauss}^2 = 13.598\ \text{ev} \quad (29)$$

$$\frac{m_e^3 B_a^2}{4\pi m^3 r \omega^2} = 1.097373 \times 10^5 \frac{\text{waves}}{\text{cm.}}$$

$$= 3.54709 \times 10^{-10} \frac{\text{gauss}^2\ \text{sec.}^2}{\text{cm.}} = \text{Rydberg constant} \quad (30)$$

$$r^3 \omega B_a = 2.53258 \times 10^8 \frac{\text{cm.}^3}{\text{sec.}^2}$$

$$= 1.43987 \times 10^1 \frac{\text{cm.}^3\ \text{gauss}}{\text{sec.}} = \frac{e^2}{\epsilon_0 m_e} \quad (31)$$

$$\frac{m\omega}{B_a} = 1.24818 \times 10^{-25}\ \text{g.} = 2.19542 \times 10^{-18} \frac{\text{g.}}{\text{sec.}\cdot\text{gauss}} = \frac{m_e}{\alpha_e} \quad (32)$$

$$m r^3 B_a = 2.97124 \times 10^{-40} \frac{\text{g. cm.}^3}{\text{sec.}}$$

$$= 1.68926 \times 10^{-47}\ \text{g. cm.}^3\ \text{gauss} = \frac{\hbar^3 \alpha_e}{m_e c} \quad (33)$$

$$\frac{m r^2 \omega^3}{B_a} = 1.12181 \times 10^{-4} \frac{\text{g. cm.}^2}{\text{sec.}^2}$$

$$= 1.97315 \times 10^3 \frac{\text{g. cm.}^2}{\text{gauss sec.}^3} = \frac{m_e c^2}{\alpha_e}. \quad (34)$$

Note that quantity 25 is equal to ϵ_0, as proposed previously (7, 8).

Quantity 26 is the well-known fine structure constant of the electron. Quantity 27 is equal to the square of electronic charge, that is, equal to e^2. Quantities 28, 29 and 30 will be recognized as the first Bohr radius, the Rydberg energy unit, and the Rydberg constant, respectively. Quantity 33 will be observed as equal to the product, $2m_eM_B$, that is, electronic mass times twice the Bohr magneton of magnetic moment.

Note also that the four constants 31 through 34, as well as the constants 19 and 20, do not require electronic rest mass m_e in their derivations. This suggests a *general* quality of these quantities, as opposed to the more specific electronic character of constants 21 through 30 which do require m_e for derivation.

Observed interrelationships of quantities 33, 21 and 19 might suggest an important *general* significance to the equality, $L_m = m_eM_BS_e^{-1}$, where S_e is electronic spin $\hbar/2$. "Spin" may then be somewhat generally defined as a quantity S equal to mass·magnetic moment·L_m^{-1}.

Thus, some twenty-nine well-known constants (6 through 34) may be surveyed to characterize or "prove" the five universal units (of mass, space, time, temperature and magnetic intensity) as common denominators of many fundamental quantities and, likely, of the most fundamental laws of nature. Such accurate numerical and dimensional common denominators might then be supposed to have potential value in the search for yet undisclosed constants and/or laws of particle physics. For instance, it may be that the following constants have value for correlations of phenomena in the physics of astronomic bodies:

$$\frac{rB_a}{m\omega} = 1.2943 \times 10^{-8}\ \frac{\text{cm.}}{\text{g.}}$$

$$= 7.3586 \times 10^{-16}\ \frac{\text{cm. sec. gauss}}{\text{g.}} = \mu_b = \left(\frac{G\hbar\alpha_e^2}{m_e^2c^3}\right)^{1/2} \quad (35)$$

$$\frac{mr\omega^4}{B_a^2} = 3.981056 \times 10^{8}\ \frac{\text{g. cm.}}{\text{sec.}^2}$$

$$= 1.23163 \times 10^{23}\ \frac{\text{g. cm.}}{\text{sec.}^4\ \text{gauss}^2} = F_g = \frac{m_e^2c^3}{\hbar\alpha_e^2} = \frac{G}{\mu_b^2}. \quad (36)$$

As suggested in the formula for constant 35, that is, μ_b, a (magnetic permeability[3]) constant of proportionality may be postulated to exist between the product of a rigid body's (stellar, solar, or terrestrial) radius (r, in cm.) times its magnetic field intensity (B_a, in sec.$^{-1}$ units) and the product of its mass (m, in g.) times its angular velocity (ω, in rad·sec.$^{-1}$). Such a constancy will be observed as qualitatively equivalent to a constant ratio of magnetic moment (r^3B) to angular momentum ($mr^2\omega$), which reiterates an often-recurring hypothesis (9) regarding the source of terrestrial and stellar magnetic fields.

[3] Note the similarity of 35 to the constant 22.

The constant 36, that is, F_9, is then simply a correlation $G\mu_b^{-2}$ of the constants of gravitational interaction and magnetic permeability. Since F_9 may also be shown to correlate quantum mechanical and electromagnetic quantities in the forms $\hbar c L_m^{-2}$, or $m_e^2c^3(\alpha_e^2\hbar)^{-1}$, or $\epsilon_0^2\hbar m_e^2c^3e^{-4}$, it might be considered as a possible candidate for the long-sought general constant and/or law of both astronomic and submicroscopic particle physics. Evidence for such a generality or unification, as previously suggested (7, 8), appears to be inherent in the equation $\hbar c\mu_b^2/GL_m^2 = 1.000$.

If the constants 35 and 36 should eventually become established as highly pertinent fundamental quantities, for example, as with G, $\hbar$, c, etc., it would then be evident that the universal units 1 through 5 might define many additional constants of possible value to astronomic or cosmological theory, for example,

$$\frac{r^3B_a}{m^2} = 2.880 \times 10^{-26}\ \frac{\text{cm.}^3}{\text{sec. g.}^2} = \frac{G\mu_b}{c} = \left(\frac{G^3\hbar\alpha_e^2}{m_e^2c^5}\right)^{1/2} \tag{37}$$

$$\frac{r^2B_a}{m} = 3.880 \times 10^{2}\ \frac{\text{cm.}^2}{\text{sec. g.}} = \mu_bC = \left(\frac{G\hbar\alpha_e^2}{m_e^2c}\right)^{1/2} \tag{38}$$

$$\frac{r^3\omega B_a}{m} = 1.163 \times 10^{13}\ \frac{\text{cm.}^3}{\text{sec.}^2\text{ g.}} = \mu_bc^2 = \left(\frac{G\hbar c\alpha_e^2}{m_e^2}\right)^{1/2} \tag{39}$$

$$\frac{r^2\omega^3}{B_a} = 5.1534 \times 10^{6}\ \frac{\text{cm.}^2}{\text{sec.}^2} = \frac{G}{\mu_b} = \left(\frac{G^3\hbar\alpha_e^2}{m_e^2c^3}\right)^{1/2} \tag{40}$$

$$\frac{mr^2\omega^5}{B_a^2} = 1.194 \times 10^{19}\ \frac{\text{g. cm.}^2}{\text{sec.}^3} = \frac{Gc}{\mu_b^2} = \frac{m_e^2c^4}{\hbar\alpha_e^2} \tag{41}$$

$$mB_a = 7.045 \times 10^{58}\ \frac{\text{g.}}{\text{sec.}} = \frac{\mu_bc^5}{G^2} = \left(\frac{\hbar c^7\alpha_e^2}{G^2m_e^2}\right)^{1/2} \tag{42}$$

$$rB_a = 5.2284 \times 10^{30}\ \frac{\text{cm.}}{\text{sec.}} = \frac{\mu_bc^3}{G} = \left(\frac{\hbar c^3\alpha_e^2}{Gm_e^2}\right)^{1/2} \tag{43}$$

$$\frac{B_a}{\omega} = 1.744 \times 10^{20} = \frac{\mu_bc^2}{G} = \left(\frac{\hbar c\alpha_e^2}{Gm_e^2}\right)^{1/2}. \tag{44}$$

As noted previously, the constants 37 through 44 can also be alternately expressed in terms of gauss units by substituting B for B_a in the formulae. Each can be rather precisely derived from well-known and accepted constants as illustrated in the above equations. The square of the constant 44, that is, $(\mu_bc^2/G)^2 = \hbar c\alpha_e^2/Gm_e^2 = e^2\alpha_e/Gm_e^2\epsilon_0 = B_a^2\omega^{-2} = 3.04154 \times 10^{40}$ may be considered to rep-

resent a relatively exact value of the dimensionless cosmological constant discussed by Bondi (5) and Zimmerman (6). It may then be significant to observe that Hubble's constant, that is, 540 km.·sec.$^{-1}$ megaparsec^{-1} or 1.75×10^{-17} sec.$^{-1}$, is equal to

$$\frac{5\omega^4}{B_a^3} = \frac{1.7491 \times 10^{-17}}{\text{sec.}} = \frac{5Gm_e^3c^3\epsilon_0^2}{e^4\alpha_e} = \frac{5Gm_e^3c^2}{\hbar\alpha_e^3}. \tag{45}$$

Thus, many of the fundamental physical constants, which are vital to the interpretation of phenomena in quantum mechanics, electromagnetism, magnetism, atomic theory, thermodynamics, classical mechanics and cosmology, appear to depend on perspicuous interrelationships of the five simple constants:

$$m = 2.17699 \times 10^{-5} \text{ g.} = (\hbar cG^{-1})^{1/2} \tag{1a}$$

$$r = 1.61562 \times 10^{-33} \text{ cm.} = (G\hbar c^{-3})^{1/2} \tag{2a}$$

$$\omega = 1.85559 \times 10^{43} \text{ rad}\cdot\text{sec.}^{-1} = (c^5/G\hbar)^{1/2} \tag{3a}$$

$$t = 5.38912 \times 10^{-44} \text{ sec.} = (G\hbar c^{-5})^{1/2}$$

$$T = 7.08681 \times 10^{31} \text{ °A} = (\hbar c^5/4Gk^2)^{1/2} \tag{4a}$$

$$B = 1.84253 \times 10^{56} \text{ gauss} = 2m_eM_Bc^4/G\hbar^2 \tag{5a}$$

$$(B_a = 3.23640 \times 10^{63} \text{ sec.}^{-1} = 2m_eM_Bc^4/G\hbar^2 = c^3\alpha_e/Gm_e).$$

Note in the formula for B that the Bohr magneton M_B has its conventional value (constant 21) of 9.28370×10^{-21} cm.^{3}·gauss; while for B_a it has its alternate postulated (7, 8) value (of 1.63291×10^{-13} cm.^{3} sec.$^{-1}$ $= \hbar^2\alpha_e/2m_e^2c$.

If the five quantities are employed as new "universal units" of measure for mass, space, time, temperature and magnetic intensity, it will be noted that most of the previously discussed universal constants, such as c, $\hbar$, G, L_m, μ_b, F_g, etc., would acquire coefficients of unity, that is, 1.000 rt^{-1}, 1.000 mr^2t^{-1}, 1.000 $r^3t^{-2}m^{-1}$, etc. Since certain of their coefficients involve impractical exponents, it is unlikely that such units would ever be adopted directly into the systems of measurement. However, it may not be inconceivable eventually to consider adoption of modified units such as

m = 217.699 g. = 1.0 "mar" = mass unit

l = 1.61562 cm. = 1.0 "lene" = length unit

t = 0.538912 sec. = 1.0 "tine" = time unit

°T = 0.0708681 °A = 1.0° "Tem" = temperature unit

M = 1.84253 gauss = 1.0 "mint" = magnetic intensity unit
(M_a = 3.23640 × 10^7 sec.$^{-1}$ = 1.0 "mint" = 1.744 × 10^7 tine^{-1} = alt.)

Though the particular choice of magnitudes and names is not of essential importance, it will be recognized on inspection that the given numerical values of the coefficients might afford inherent advantages not characteristic of the many random units selected so arbitrarily in the past.

In truth, the primary incentive for investigations of "universal units" has little to do with the simple selection of such new numerical values or dimensions, which might only confuse an already preposterous situation in the several systems of physical measurements. Rather, the appropriate goal of such investigations should be the possible revelation of additional fundamental knowledge of physical phenomena. For instance, as previously proposed (**7, 8**) if the constant L_m (quantity 19) should precisely embody the essence of Dr. Heisenberg's (**10**) postulated "constant of length" (about 10^{-13} cm.), or if the constant μ_b (quantity 34) should represent a lucid expression of Dr. Blackett's proportionality constant (**9**), valuable new tools would have been made available to search the remaining mysteries of nuclear forces and "new particle" properties, as well as terrestrial and stellar magnetism (and other mysterious cosmological principles). Simple-appearing numerical and/or dimensional constants such as π, G, $\hbar$, c, e, k, etc., have played almost indispensable roles in arriving at the present state of scientific knowledge and development, since they afford important natural correlations of space, time, and matter phenomena. This may lead to the conclusion that valid new constants and correlations can play similarly great roles in the future. It will be noted that, although only some forty universal constants have been illustrated in this paper, various combinations and permutations of the five given "universal units" (and their squares and cubes) can hypothetically be used to derive tens of thousands of "new" physical constants, each of which may have a pertinent value to the intimate understanding of universe structure. Their validation might then open the way to vast stores of untapped fundamental information, which would presumably be helpful in the resolution of several modern mysteries of submicroscopic and cosmological physics.

Indeed, the eventual understanding of matter and space, as well as universe structure, may depend heavily on unsuspected revelations embodied in such new constants and/or the natural laws which they prescribe.

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APPENDIX

One purpose of this paper is to suggest a quantitative basis for proof of the previously (**7, 8**) postulated equality: gauss unit = 1.7589×10^7 sec.$^{-1}$. This equality, which hints that the phenomenon of magnetic intensity is somehow a reflection of a "time" or "inverse-time" property of universe particles, may be believed to have significant fundamental importance.

The nature of non-mechanistic "word units" of electricity and magnetism, for example, the gauss, oersted, amp, volt, etc., can be considered effectively to obscure the causal bases for the phenomena which they describe or define. Granting the existence of an inherent mechanistic quality of magnetic intensity, as exemplified in the unit = 1.7589×10^7 sec.$^{-1}$, it will be recognized from other discussions (**7, 8**) that all of the above-mentioned word units can then be expressed in terms of similar mechanical equivalents. These equivalents seem to afford a direct means for a quite perspicuous mechanical analysis of each and every phenomenon of electrostatics, magnetostatics, electromagnetism, and practical electricity. Such an analysis is believed valuable for alleviating the abstract quality of such phenomena, and may indeed be a requirement for establishing the precise relationship of the magnetic fields to the motions of universe particles, for example, as previously discussed (**7, 8**) in connection with the "new" constants L_m, μ_b, and F_g. Similarly, such analysis may be a requirement to "explain causally" the mysterious space properties inherent in the magnetic permeability constant (μ_0) and the dielectric constant (ϵ_0).

The concept that the various universal constants of physics contain the key to "universal units" of measure is certainly not a novel idea. However, the various scientists who have given thought to the subject have usually had to make somewhat arbitrary choices among those constants available; and most have recognized that certain of nature's generalizations or constants have not yet been disclosed, and thus suspect that the jig-saw puzzle is not yet complete. Part of this incompleteness may seem to arise from the fact that the electronic or electromagnetic constants such as $\mu_0 = 1.0$ and $\epsilon_0 = 1.0$ have very effectively obscured the presence of missing pieces to the puzzle. Perhaps the early conclusion that the electron was somehow the only "fundamental particle," or one among a very few, could also lie at the philosophical heart of the problem. In any event, it appears that disclosure of causal bases for interpreting the phenomena and constants of electromagnetic theory is paramount to the eventual resolution of many mysteries contributing to the "crises of modern theory." It will be noted that the paradox $\epsilon_0 = 1/\mu_0 c^2$ has often created arguments and befuddled attempts to interpret abstruse interrelationships of the esu and emu systems of electricity and magnetism. An answer to this and other riddles can possibly have a major catalytic effect on the extension

of fundamental knowledge in all areas of theoretical science. Hopefully, the derivation of "universal units" and/or new universal constants can contribute effectively to such catalysis.

The observant reader will recognize that the proposed universal m, r, t, and T units vary only slightly from those which Sir Arthur Eddington proposed some forty years ago. One cannot be certain how much interest Eddington actually had in these units or how importantly they may have figured into his general feeling and approach toward fundamental matters. At the time it is likely that his contemporaries considered his speculations quite wild or without especial significance. It will be noted that where these units are considered to represent properties of some hypothetical photon, its rest mass and "frequency" would be indicative of a particle-wave with the fantastic energy of some 10^{16} ergs, or some 10^{28} times the intrinsic energy of an electron. Forty years ago it would probably have seemed unbelievable that future experimentation or measurement might eventually reveal individual cosmic rays, or particle-waves, with energies as high as 10^{4} ergs or greater! It may be possible that Eddington considered such a hypothetical 10^{16}-erg particle as an upper limit to the mass and energy scale of the "fundamental particles" or photons of submicroscopic physics, and/or perhaps as a lower limit to which the macroscopic particles of classical mechanics might be disrupted by centrifugal forces, for example, occurring on their approach to the light velocity limit.

Whatever the case, it seems that Eddington might have at least suspected that the primary physical constants of classical mechanics (G) and quantum mechanics (h), in conjunction with the light velocity constant c (believed to represent an upper limit to the velocity of either submicroscopic or macroscopic mass particles), somehow held the secret of nature's quantitative ordering of mass-space-time. If so, they would probably afford a means for unification of physical theory, which he so avidly sought.

The major twentieth century champion of the causal philosophy and theory was Albert Einstein. Although he had revolutionized the way of thinking in physics with general methods which also became fundamental for the "modern" quantum mechanics and its interpretation, he stoutly maintained until his death that even the so-called quantum-features of atomic phenomena could in principle be explained along the lines of the more "ancient" classical physics of fields. Although the concept of "physical reality" in atomic physics was mathematically generalized by Bohr's concepts of complementarity, in such a way that the experimental arrangement was considered to be an essential part of the theoretically described phenomena, Einstein still wanted to keep the ideal of classical celestial mechanics—that the objective physical state of a system must be entirely independent of the way in which it is observed. Although he frankly admitted that his hopes for a total solution on such classical lines had not yet indeed been fulfilled—and the possibility of such a theory had not yet been proved by him—he still considered this to be an open question, much to the chagrin of his many "modern" adversaries. When Einstein spoke of a "unified field theory," he had in mind a theory which would solve all problems regarding the elementary particles of matter with the help of classical fields which were everywhere regular, that is, free of singularities.

It may be significant that Dr. Einstein once attributed significance to a universe particle characterized by properties approximately equal to: $m = 10^{53}$ to 10^{54} g., $r = 2 \times 10^{27}$ cm., and $t = 6 \times 10^{16}$ sec. These quantities seemed to be pertinent on the basis of Hubble's constant, the age of the universe, and the apparent mean density of matter in the universe. It will be noted that they very roughly approximate the values for m, r, and ω^{-1} of one of the "particles" (No. IV) listed in Table I. Similarly, the m, r, and ω^{-1} values for particles II and III are close approximates to those proposed by Dirac (**2**) and Eddington (**3**), respectively. If one will observe the numerical relationships between the quantities of I and III, and V and III, it will be recognized that the hypothetical "particles" I and V are arbitrarily related to particle III by the cosmological constant, that is, 3.04154×10^{40}. (A log-log plot of the r, ω, B, and T values *versus* the m values will reveal hypothetical limits of minima in the rm^{-1}, $m\omega^{-1}$, mB^{-1}, and mT^{-1} ratios of universe particles of any mass.) It will be noted that $r\omega = c$, $m(r^3T^4)^{-1} =$ constant, and $mr\omega^4B^{-2} = F_g$, is characteristic of all five particles; $mr^2\omega = h$ and $rB\omega^{-1} = L_m$ is characteristic of particles I, II, and III; and $r^3\omega^2m^{-1} = G$ and $rB(m\omega)^{-1} = \mu_b$

TABLE I.—*Hypothetical Properties of Submicroscopic and Classical Mechanical Particles at mc^2.*

	I	II	III	IV	V
m	7.1515×10^{-46} g.	1.2477×10^{-25} g.	2.1770×10^{-5} g.	1.1561×10^{56} g.	2.0139×10^{76} g.
r	4.9173×10^{7} cm.	2.8185×10^{-12} cm.	1.6156×10^{-33} cm.	8.5792×10^{27} cm.	1.4946×10^{46} cm.
ω	6.0962×10^{2} sec.$^{-1}$	1.0636×10^{23} sec.$^{-1}$	1.8556×10^{42} sec.$^{-1}$	3.4942×10^{-18} sec.$^{-1}$	2.0058×10^{-38} sec.$^{-1}$
B	3.4970×10^{-18} sec.$^{-1}$	1.0636×10^{23} sec.$^{-1}$	3.2364×10^{42} sec.$^{-1}$	6.0962×10^{2} sec.$^{-1}$	3.4970×10^{-18} sec.$^{-1}$
T	2.330×10^{-9} °A.	4.063×10^{11} °A.	7.0868×10^{21} °A.	3.077×10^{1} °A.	2.330×10^{-9} °A.

m = mass, in g.; r = equatorial radius, in cm.; ω = angular velocity, in radians·sec.$^{-1}$; B = magnetic intensity, in sec.$^{-1}$ units; T = "effective particle temperature," in °Absolute.

TABLE II.—*Calculated Polar Intensities for Magnetic Fields of Astronomic Bodies.*

Astronomic Body	Mass g. (M)	Axial Radius cm.·rad^{-1} (r_a)	Angular Velocity rad.sec.$^{-1}$ (ω_r)	Polar Magnetic Intensity sec.$^{-1}$* (B_g)	Polar Magnetic Intensity gauss* (B_g)	Blackett's Values gauss (B_b)
B.0 Star	2.96×10^{34}	4.8×10^{11}	2.18×10^{-5}	1.74×10^{10}	990	500
B.5 Star	1.18×10^{34}	3.0×10^{11}	3.54×10^{-5}	1.8×10^{10}	1023	500
A.0 Star	5.32×10^{33}	2.3×10^{11}	6.8×10^{-5}	2.04×10^{10}	1160	800
78 Virginius	4.6×10^{33}	1.4×10^{11}	7.3×10^{-5}	3.11×10^{10}	1770	1500
A.5 Star	3.54×10^{33}	1.11×10^{11}	9.74×10^{-5}	4.02×10^{10}	2282	1200
F.0 Star	2.95×10^{33}	9.76×10^{10}	1.06×10^{-4}	4.15×10^{10}	2360	1700
Sun	1.97×10^{33}	6.95×10^{10}	2.95×10^{-6}	1.06×10^{9}	60	53
Jupiter	1.9×10^{30}	7.0×10^{9}	1.75×10^{-4}	6.15×10^{8}	35	—
Saturn	5.68×10^{29}	5.8×10^{9}	1.7×10^{-4}	2.16×10^{8}	12.3	—
Neptune	1.03×10^{29}	2.66×10^{9}	1.1×10^{-4}	5.52×10^{7}	3.14	—
Uranus	8.73×10^{28}	2.55×10^{9}	1.63×10^{-4}	7.24×10^{7}	4.11	—
Earth	5.98×10^{27}	6.36×10^{8}	7.29×10^{-5}	8.86×10^{6}	0.51	0.61
Venus	4.9×10^{27}	6.1×10^{8}	2.43×10^{-6}	2.53×10^{5}	0.014	—
Mars	6.46×10^{26}	3.4×10^{8}	7.3×10^{-5}	1.8×10^{6}	0.102	—
Mercury	3.35×10^{26}	2.42×10^{8}	8.28×10^{-7}	1.49×10^{4}	0.00085	—
Moon	7.36×10^{25}	1.735×10^{8}	2.66×10^{-6}	1.47×10^{4}	0.00084	—

$\mu_b = \dfrac{r_a B_g}{M \omega_r} = 1.295 \times 10^{-8} \dfrac{\text{cm.}}{\text{g.}}$; $B_g = \mu_b M \omega_r r_a^{-1}$.

* Gauss = 1.7594×10^{7} sec.$^{-1}$ = maxwell·cm.$^{-2}$.

is characteristic of particles III, IV, and V. Thus, in general, it may be considered that mass particles intermediate between III and V may be categorized as "macroscopic or classical mechanical" particles, whereas those with masses between I and III would be categorized as "submicroscopic or quantum mechanical" particles. (Though perhaps of no practical significance, the minima and maxima prescribed in a plot of the quantities for I–V may afford interesting philosophical speculation regarding the structural limits of universe particle systems, for example possibly regarding matter under conditions varying from its highest density state (particle III) to its lowest density state (particles I and V).) However, for the purposes of this discussion, the presentation is intended to serve only as a means of relating the "universal units," as chosen in this study, to some which have been proposed in the past. It also affords a means of suggesting applications for universal constants such as those defined in quantities 6, 9, 10, 12, 14, 16, 42, 43, and 44.

One of the main arguments which can be raised against the choice of the given universal unit for magnetic intensity (and/or the proposed universal constant μ_0) will probably involve various uncertainties and the unproven status of Blackett's original hypothesis. Table II is presented to indicate physical data (for various stars, the sun, the planets, and the moon) which permit calculations of magnetic intensities by use of the constant μ_0. Blackett's (9) correlations and magnetic intensity data (last column of Table II) were arrived at by measurements of the solar and stellar magnetic intensities by the Zeeman effect. Since the time he originated the hypothesis of a proportionality constant, other inconsistent Zeeman measurements of the solar field and some of the stellar fields have created doubt concerning the generalizations he proposed. It is argued that apparent cyclic and local variations in such fields prevent a knowledgeable estimate of the general field characterizing the stellar bodies.

In regard to these arguments, it may be suggested (from the fact that μ_0 appears as the square in the generalization $F_g = G\mu_0^{-2}$) that cyclic or seasonal variations in such fields might well result from instantaneous interactions of a body's field with some related satellite or mass-center, as previously (8) indicated. Also, since quantity 23 in this paper (Zeeman's displacement constant $= B(4\pi m_e\omega^2)^{-1} = 4.66885 \times 10^{-5}$ gauss sec.^{2} g.$^{-1}$ or 4.66885×10^{-5}/cm·oersted) was observed to be specifically an electronic quantity, that is, dependent on electronic mass m_e for its derivation, it might be surmised that intensity measurements of stars by the Zeeman effect could actually be in error—or at least considerably dependent on given electronic properties of the stars. Since it may now be believed that magnetism is not exclusively a property or phenomenon of electrons (for example, from evidence of neutron and nucleon magnetic moments), it is possible to believe that a more *general* constant than quantity 23 is needed to evaluate the magnetism of celestial bodies.

It will be noticed that Table II includes calculations of fields on the planets and the moon, that is, where no estimates or measurements by the Zeeman effect have yet been made. These values may then be considered as hypotheses or predictions, which may someday be subject to verification by direct magnetometer measurements from space-satellites or missiles.

For the purposes of this paper, such discussions of the Zeeman constant are presented primarily to illustrate the possibility that knowledge of the given "universal units" will afford a method for perspicuously characterizing physical constants as either "universal" constants or as "specific particle" constants (for example, which correlate phenomena of a single particle system).

In a broader sense, it can be hoped that the "universal units" may be aptly used (in conjunction with causal interpretations of the universal constants such as c, $\hbar$, L_m, G, μ_0, and F_g) to test eventually Einstein's belief that causal models based along the lines of celestial mechanics, can ultimately "explain" complex phenomena in the realm of submicroscopic particle physics.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

October 21, 1959

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in Franklin Hall, following the Annual Medal Day Reception and Dinner at which 420 members and guests were present.

Wynn Laurence LePage, President, called the meeting to order and announced that the minutes of the May Stated Meeting had been printed in the June issue of the JOURNAL OF THE FRANKLIN INSTITUTE. There being no corrections or additions, the minutes were approved as published.

The President stated that for many years it has been the custom of the Board of Managers of the Institute to propose the Franklin Medalist for election to Honorary Membership in The Franklin Institute, and at the Stated Meeting on Wednesday, May 20, 1959, the membership of the Institute elected Dr. Hans Albrecht Bethe to honorary membership in The Franklin Institute effective October 1959. Mr. LePage then presented to Dr. Bethe the Certificate of Honorary Membership in The Franklin Institute of the State of Pennsylvania.

Mr. LePage then asked Dr. John S. Burlew, Executive Vice President, to review plans for the coming year in the Institute.

As an introduction to the presentation of the Medal Awards, Mr. LePage spoke of events which have happened in the world in the past year and stated that the members and guests were gathered in peaceful assembly to recognize, without regard to nation or political ideology, men of good will whose efforts have been for the good of all mankind. He characterized the Medalists as men who have helped push back the curtains of ignorance and darkness and who have helped us take one more step toward a full knowledge of this universe we live in.

The President then recognized the Sponsors who presented the recipients of the Awards, and the presentation of the Medals was made by President LePage.

Following the presentation of Medals, President LePage presented the speaker, Dr. Hans Albrecht Bethe, Franklin Medalist, and Director of the Laboratory of Nuclear Studies at Cornell University, Ithaca, N. Y. Dr. Bethe gave a most interesting and thought-provoking talk on "Fundamental Particles."

President LePage expressed warm and sincere thanks to Dr. Bethe for his excellent talk, and thanked all for a full and wonderful evening.

The meeting was adjourned at 10:10 P.M.

WILLIAM F. JACKSON, JR.
Secretary

NOTE: The entire proceedings of Medal Day will be published in the December issue of the JOURNAL.

MEDAL DAY PROGRAM

Reception for Medalists	The Hostess Committee
Invocation	The Reverend Ernest A. Harding, D.D.
Dinner	Medalists, Officials and Guests
Toast to Benjamin Franklin	Wynn Laurence LePage, *President*
Introduction of Former Medalists	William G. Schmidt, *Chairman*, Committee on Science and the Arts
Stated Meeting of the Institute	The President presiding
Coming Events at the Institute	John S. Burlew, *Executive Vice President*
Presentation of Medalists	Committee on Science and the Arts
Award of Medals	The President
"Fundamental Particles"	Hans Albrecht Bethe, *Franklin Medalist*

1959 MEDALS

Cresson Medals (1848)

JOHN HAYS HAMMOND, JR.
President and Treasurer
Hammond Research Corporation
Gloucester, Massachusetts

Remote Radio Control of Moving Vehicles

HENRY C. HARRISON
Consulting Engineer
Port Washington, New York

Matched Impedance Principle in Electromechanics

IRVING WOLFF
Vice-President, Research
Radio Corporation of America Laboratories
Princeton, New Jersey

Contributions to Radio, Radar and Electronics

Longstreth Medals (1890)

DAVID M. POTTER
President
Potter Aeronautical Corporation
Union, New Jersey

Balanced Rotary Flow Meter

JACOB RABINOW
President
Rabinow Engineering Company, Inc.
Washington, D. C.

Magnetic Fluid Clutch

Potts Medal (1906)

GEORGE W. MOREY
Chemist
United States Geological Survey
Washington, D. C.

Rare Earth Oxide Glass

Henderson Medal (1924)

ELECTRO-MOTIVE DIVISION
GENERAL MOTORS CORPORATION
LaGrange, Illinois
Received by
Cyrus R. Osborn, Vice-President,
General Motors Corporation

Diesel-electric Locomotives

Wetherill Medals (1925)

ROBERT B. AITCHISON
Consultant
Linde Company, Division of Union Carbide Corporation
New York, New York

Jet-Piercing Process for Mining and Quarrying

CLARENCE ZENER
Director, Research Laboratories
Westinghouse Electric Corporation
Pittsburgh, Pennsylvania

Contributions to the Field of Solid State Physics

Wetherill Medals (cont.)

A. T. JAMES
Research Biochemist
National Institute for Medical Research
London, England

A. J. P. MARTIN
Director
Abbotsbury Laboratories, Limited
Elstree, Hertfordshire, England

R. L. M. SYNGE
Research Biochemist
Rowett Research Institute
Bucksburn, Aberdeenshire, Scotland

Gas Chromatography

Clark Medal (1926)

GEORGE G. OBERFELL
Consultant
Phillips Petroleum Company
Bartlesville, Oklahoma

Liquid Petroleum Gas Development for Domestic Fuel

Brown Medal (1938)

HARDY CROSS
Professor and Author
Virginia Beach, Virginia
(Awarded Posthumously)

Contributions to Structural Engineering

Clamer Medal (1943)

MORRIS COHEN
Professor, Physical Metallurgy
Massachusetts Institute of Technology
Cambridge, Massachusetts

Contributions to Science and Practice of Physical Metallurgy

Ballantine Medals (1946)

ALBERT HOYT TAYLOR
Formerly Chief Consultant
United States Naval Research Laboratory
Washington, D. C.

Contributions to the Science and Technology of Electronic Communications

CHARLES H. TOWNES
Professor, Physics
Columbia Radiation Laboratory
Columbia University
New York, New York

Conception and Development of the MASER

The Franklin Medal (1914)

HANS ALBRECHT BETHE
Professor, Laboratory of Nuclear Studies
Cornell University
Ithaca, New York

Contributions to Theoretical Physics

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MEMBERSHIP

The following elections were approved on October 1, 1959 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

Stanley L. Albright
Thomas Bain, Jr.
Robert W. Bardsley
Allen Baxter
J. Robert Behrman
Robert D. Bent
Elizabeth W. Bingham
Frances Brown
Jasper M. Brown, Jr.
Gouverneur Cadwalader, Jr.
David J. Constantine
J. Frederick Cox, 2nd
Kenneth M. Curto
Daniel J. Duffin
A. Denise Fowler
Kenneth W. Goff
Hugh M. Hamill
Richard J. Hanak
Clement Handlon
John F. Hirst
Herman Hoffman
Karl R. Kurz
Mrs. Philler Lee
William E. Lunt, Jr.
Pierre F. P. Martin
John A. Mathe
James M. McBride, Jr.
David F. McLaughlin
Frederick G. McMullen
Conrad Melas
Stewart Miller
William J. Mitchell, Jr.
Ronald C. Moore
Frederick J. Moury
Arthur L. Mueller
Ralph J. Myers
Rosalind Nadell
John J. Nothelfer
Louis Ollivier
William J. O'Meara
Samuel A. Ott
Erwin H. Panczner
Alton L. Penniman
William E. Purdy
Carlo E. Rossi
John J. Serrell
James John Silcox
Edward R. Stroik
Edmund Thayer, Jr.
John E. Wagenseller
James A. Webb
Walter Weinstock

EDUCATOR

Abner A. Miller
K. F. O'Driscoll
Philip R. Trommer

NON-RESIDENT

Robert E. Ross

NECROLOGY

Wallace E. Belcher '42
William W. Bodine '36
Ronald M. Heebner '56
Frederick P. Ristine '49

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, October 14, 1959.)

HALL OF THE COMMITTEE,
PHILADELPHIA, OCTOBER 14, 1959.

DR. WILLIAM G. SCHMIDT in the Chair.

The following reports were presented for final action:

No. 3298: Membrane Demineralization.

This report recommended the award of a John Price Wetherill Medal to Walter Juda, of Cambridge, Massachusetts, "For his development of a commercial method of desalting saline water by electrodialysis."

No. 3363: The Newcomen Medal.

This report recommended the award of The Newcomen Medal to Clarence Barton Campbell, of Swarthmore, Pennsylvania, "In consideration of his many outstanding achievements in the utilization of Steam for the benefit of mankind."

D. S. FAHRNEY,
Secretary to Committee

BOOK REVIEWS

EXPERIMENTAL NUCLEAR PHYSICS, Volume III, edited by E. Segrè. 811 pages, illustrations, $6 \times 9\frac{1}{4}$ in. New York, John Wiley & Sons, Inc., 1959. Price, $23.00.

This final volume of *Experimental Nuclear Physics* lives up to the high standards of authoritative contributions set by the first two volumes which were published in 1953. Since it seems desirable that such a series of survey articles describe the status of the different aspects of the subject at approximately the same time, the long delay in the completion of the third volume is unfortunate as is the fact that some of the reviews in it were written as early as 1955 and were only slightly revised while others were re-written.

In the first chapter, E. Segrè (Berkeley) summarizes the laws of radioactive decay and discusses briefly the methods which have been developed for the measurement of decay constants. A useful summary of dosimetry, tables of radioactive standards and assaying methods and a discussion of the statistical fluctuations encountered with a finite number of decaying atoms round out the chapter.

The study of the transuranic elements and the search for new heavy elements have given a large impetus to the field of alpha radioactivity. The present status of the experimental techniques, the vast amount of information obtained with them and its significance for our knowledge of the nucleus are comprehensively summarized by G. C. Hanna (Atomic Energy of Canada Limited). Since reviews on alpha radioactivity do not seem to be as abundant as those on the other types of nuclear radiation, it is fortunate that the author wrote an up-to-date version which gives an account of the role played by the different nuclear models in the interpretation of alpha particle spectra.

The subsequent chapters on gamma rays and on beta rays were jointly written by M. Deutsch (M.I.T.) and O. Kofoed-Hansen (Denmark) in 1955 and give a good account of the techniques and theory at that time. The presentation is more concentrated than that in Siegbahn's book on beta- and gamma-ray spectroscopy which was published in 1955, and therefore, covers approximately the same material. With the exception of the experiments demonstrating and studying the non-conservation of parity in beta decay, recent developments are only dealt with by references to existing survey articles. The "parity-experiments" are discussed as part of a very readable section on experiments concerning neutrinos.

In the last chapter, the history of particle accelerators is very ably told by one of the "insiders" (E. McMillan, Berkeley) and is documented with many photographs and drawings. Principles of operation, conditions for orbit stability, etc., are clearly developed, and the reader participates in some of the anxieties of the designer of the tools of modern physics.

While it cannot be said that the third volume of Experimental Nuclear Physics fills an "urgent need," it contains enough new and conveniently available information to make its possession a must for any institution interested in nuclear physics. The individual scientist might take a second look at the price tag!

F. R. METZGER
Bartol Research Foundation

SOLID STATE PHYSICS, Vol. 8, edited by F. Seitz and D. Turnbull. 519 pages, illustrations, 6×9 in. New York, Academic Press, 1959. Price, $13.50.

The recently published eighth volume of *Solid State Physics* meets the same high standards as the preceding volumes. Since most of the basic and background topics have been covered in the early volumes, this book is entirely devoted to specialized topics.

The first topic by D. S. McClure (about 50 pages) deals with the "Electronic Spectra of Molecular Crystals." A second article by the same author, dealing with the electronic spectra of inorganic crystals will appear in Vol. IX. After an excellent theoretical in-

troduction based mostly on group theoretical considerations, the author reviews some selected experimental results with particular emphasis on fluorescence.

The photoconductivity of germanium (about 50 pages) is analyzed in the second article by R. Newman and W. W. Tyler. The electronic structure of Ge and photoconductivity theory are first considered, and followed by a well written exposé of the experimental aspects and a special section on Ge as an infrared detector.

In the third article, "Interaction of Thermal Neutrons with Solids," L. S. Kothari and K. S. Singwi deal chiefly with the theoretical aspects of these interactions. An excellent treatment of the general theory of elastic and inelastic neutron scattering is followed by a consideration of the slowing down of neutrons near thermal equilibrium. Many physicists, especially working in other fields, will find this review very useful.

The fourth article (about 110 pages), "Electronic Processes in Zinc Oxide," by A. Heiland, E. Mollus and F. Stockmann, is a methodical and exhaustive study of this subject. Without going into the details, let us stress that the implications of this remarkable review shed considerable light on the electronic processes on a larger class of materials.

"The Structure and Properties of Grain Boundaries" by S. Amelinckx and W. Dekeyser constitute the subject of the last article (about 180 pages). After a review of the geometry of grain boundaries, a rich collection of photographs, electron micrographs and illustrative diagrams is presented and analyzed. Various aspects of the physics of grain boundaries (diffusion along, melting at, migration of, electrical properties etc.) are successively considered. The editors of this series are to be congratulated for including in this volume such an impressive study of the "imperfect" crystal.

HENRI AMAR
The Franklin Institute Laboratories

ANALYSIS OF STRAIGHT LINE DATA, by Forman S. Acton. 267 pages, diagrams, 5¼ × 9 in. New York, John Wiley & Sons., 1959. Price, $9.00.

Perhaps the most widely used method of data analysis and presentation in research and industry is the straight line graph. In his text, Dr. Acton has thoroughly explored and exposed this analytical procedure, beginning with the general model where both the slope and intercept are unknown. The author then proceeds to other models often forgotten in practice. These are (*a*) where the slope is known with certainty, (*b*) where the intercept is fixed, (*c*) where both the intercept and the slope are known and (*d*) where the line must pass through a fixed point. With each of these models Dr. Acton carefully leads the reader through the proper analysis including such prodecures as graphical techniques and some of Tukey's "quick and non-normal" methods. The influence of the author's fellow professor, Dr. Tukey, is also felt in the section dealing with relationships where both variables are in error.

Since not all data happily fall into straight lines, the author has included a chapter on methods of transformation to linearize as well as a section on the use of orthogonal polynomials for computational ease.

With this book, the author has filled a void in applied statistics and he is to be congratulated for it. But he has done his task with such comprehension and such clarity that his text should find its way to the hands of every practicing scientist and statistician.

F. X. MCLAUGHLIN
Franklin Institute Computing Center

SERVOMÉCANISMES: THÉORIE ET TECHNOLOGIE, by M. Bonamy. 284 pages, diagrams, 6¼ × 9¼ in. Paris, Masson et Cie, 1957. Price, 4,200 fr.

Professor Bonamy has undertaken the difficult task of summarizing current servo theory and practice in a single book (it is really two books), as well as the philosophical basis of automatic-control theory in information theory and cybernetics. In the introduction he begins with a brief outline of the history of automatic control, stressing the contributions of electrical engineering, and then discusses the industrial and military needs—faster response, greater accuracy, and higher reliability—which are causing man to be replaced by feedback controllers. He then considers the social and moral aspect of automatization, concluding that it will free man from the unimaginative industrial task of

machine-watching while at the same time greatly raising his standard of living and improving the intellectual and moral climate of society. Next he qualitatively discusses servomechanisms from the viewpoint of cybernetics, using de Broglie's definition of this science: "The science of the relations between and the control and transmission of information." Finally he considers feedback control in man and in societies.

The first half of the book, "Servomechanism Theory," begins with the usual discussion of the properties of linear systems. Analogies between electrical, mechanical, hydraulic, and thermal systems are then treated from the point of view of dimensional analysis. This is followed by a semi-quantitative discussion of the desired characteristics of servomechanisms and the effects of feedback and bandpass on these characteristics.

The author then considers general analytical methods for linear systems, beginning with the classical method of solving linear differential equations with constant coefficients. Next he presents a brief discussion of the Laplace transformation, introduces the notions of generalized impedance and transfer function, and calculates the transfer functions of a number of servo elements.

The section on steady-state analysis begins with a discussion of Bode's relation between attenuation and phase in minimum-phase systems and the construction of Bode plots. The connection between steady-state and transient response is considered, with a brief presentation of the Fourier transformation. This is followed by a discussion of the Nyquist stability criterion and the Bode statement thereof, after which computational aids such as Nichols charts and plots of constant closed-loop gain and phase are presented. Finally the stability of multiloop systems is treated.

A good section on compensation follows, reflecting perhaps the author's communications engineering background. In addition to the usual lead, lag, and lead-lag networks, Professor Bonamy discusses four-pole theory and its application to the design of more complex equalizing networks, a topic too often omitted in texts on servomechanisms. The design and comparison of both series and feedback compensation for a simple position control system is given as an example in this section.

Finally a brief discussion of the describing-function method for studying nonlinear servomechanisms is presented, together with a two-page treatment of sampled-data systems.

The second half of the book, "Servomechanism Elements," is a semi-quantitative discussion of control system components and their performance. Transducers, comparators, amplifiers and modulators, relays and thyratrons, electric motors, and hydraulic and pneumatic elements are presented in that order.

The principal fault of *Servomécanismes: Théorie et Technologie* is that too much has been covered too briefly. In the reviewers' opinion it is not suitable as a text because it does not adequately cover the essential tools for the analysis of linear systems: the Fourier and Laplace transformations. It is likewise not suitable as a handbook for the practical design of control systems. In addition it does not cover more recent developments such as the analytical design of linear systems (Wiener minimization), the phase-plane method and other techniques for the analysis of nonlinear systems, and digital servo systems.

Y. H. KU AND J. H. DIETZ
University of Pennsylvania

ELEMENTS OF SOLID STATE THEORY, by G. H. Wannier. 270 pages, diagrams, 5¼ × 8¼ in. New York, Cambridge University Press, 1959. Price, $6.50.

This new contribution to the rapidly expanding literature of solid state physics, "aimed at advance students and scientists with good all-round knowledge," strikes one first by the number of topics completely or almost completely omitted. The omitted topics include diffusion and resonance phenomena, elastic and surface properties of solids, dislocations and solid state defects in general, etc. There is no mention of experimental methods, there are very few tables and very little numerical data (except in the last chapter).

Yet, despite—or more accurately thanks to—these omissions, the author, a noted theoretician, succeeds in giving us a remarkably lucid and coherent exposé of the theory behind present-day solid state physics. The first three of eight chapters are devoted to

crystal geometry, radiation diffraction and dynamics of crystal lattices. The analyses of the first and third topics are particularly penetrating and rigorous. Let us mention in particular the discussion of point groups and space groups, the determination of unit cells, and the systematic use of group-theoretical arguments. The fourth chapter is an illuminating discussion of cooperative phenomena with emphasis on ferromagnetism and the Ising model. The electron band theory of metals and semiconductors is discussed in the following two chapters with emphasis on the general principles and assumptions, while the last chapter deals with the theory of cohesion and chemical binding. Each chapter is followed by a selective and up-to-date bibliography, and supplemented by a few carefully written problems.

But probably the most remarkable features of this little book are its unity and coherence, its rigor, the lucidity of its discussions, and a most unusual style direct, compact and clear. The omission of unessential topics and digressions, has eliminated interference with the main subject. In summary, this reviewer considers the book as profitable reading for students and scientists interested in solid state physics.

HENRI AMAR
The Franklin Institute Laboratories

HOT ORGANIC COATINGS, by Raymond B. Seymour. 233 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp. 1959. Price, $7.50.

To present a brief outline of the contents of this book, we can do no better than to plagiarize the blurb on its dust jacket: "The constitution, applications and properties of hot organic protective coatings are concisely presented here. The book contains chapters on widely used hot organic materials such as asphalt, coal tar pitch, petroleum waxes and cellulose derivatives. Specific information on formulations of proprietary products is included. Additional chapters deal with hot melt applications without solvent such as peel coatings, protective linings, flame spraying and the fluidized bed process. One chapter on hot applied coal tar pitch base coatings is supplied by George B. McComb, consultant to the leading suppliers of pipe line coatings. Hot spray techniques and the many advantages of this application are also covered. This book will be helpful to all those using these coatings in any form."

It is important to the reader to realize that the author's intent, as stated in the preface, is to "maintain a broad approach and to make the presentation understandable to the average intelligent reader." In line with this purpose, Dr. Seymour states that theory has been de-emphasized in order to provide space for application data. He also states that it is not within the scope of the book to discuss the correctness of terms which are established in the coatings industry. These statements by the author probably explain such things as the references to asphalt as a "resin" and thus the implication that it is a polymer of high molecular weight. The author gives the molecular weight of asphalts as 800–1400, which is certainly not high, and I don't believe many asphalt technologists think of asphalt as a polymer.

A very difficult objective the author set for himself in the light of the diversity of his subject material is expressed as follows: "every effort has been made to ascertain that the latest materials and methods were included in this book." In accordance with this goal, he discusses on page 20 the use of loose pulverized gilsonite as a corrosion-resistant waterproof coating around hot service pipes. He does not mention that on the East Coast a pulverized hard asphalt mixed with asphalt-treated perlite is used extensively in lieu of gilsonite for this purpose.

As the reader may know, the competition between asphalt and coal tar pitch-base coatings for pipe lines is very hot. I am prejudiced in favor of asphalt because of its better mechanical properties (less tendency to crack at low temperatures or to sag at high temperatures, probably better resistance to soil stresses). This book strikes me as being heavily biased in favor of coal tar, because of its greater resistance to moisture pick-up, as illustrated on page 164, and for other reasons, too. Unfortunately, the water pick-up of both asphalts and coal tars depends considerably on variables not mentioned in the book, such as salt content, filler content, modifications of coal tar to give it better mechanical properties, etc. Blanket generalizations can be misleading.

George B. McComb, the author of Chapter 11, on hot applied coal tar pitch-base coatings, cites field tests by the American Petroleum Institute, the American Gas Association and numerous government agencies (presumably including the National Bureau of Standards). I had occasion to study the results of these tests in the late 1930's, and came to the conclusion then that they showed no significant differences in performance between coal tar and asphalt coatings of comparable thickness and number of fabric wraps. The reader is cautioned not to read into Mr. McComb's discussion of these tests any inferences to the contrary. I have discussed at length Dr. Seymour's treatment of asphalts because I am more conversant with this material than with many of the others included in the book. As far as I know, the book does indeed fulfill its purpose with regard to these. It contains many formulae and much information on a large variety of materials that I'm sure will be useful, as the preface states "for finishing, packaging and corrosion engineers . . . and other technologists and artisans."

E. THELEN
The Franklin Institute Laboratories

THE STRUCTURE OF METALS, published for The Institution of Metallurgists. 118 pages, diagrams, 5½ × 8½ in. London, Iliffe & Sons Ltd.; New York, Interscience Publishers Inc.; 1959. Price, $4.00.

This is the third volume in a series published as a result of a yearly refresher course held by the Institution of Metallurgists (London). This volume constitutes a review of the theory of the structure of metals. It contains four papers of about equal length by well known authorities in their respective fields.

The first paper by G. V. Raynor discusses the electron structure of metals. He presents a coherent picture of the developments which have taken place in this field in the last two decades. This involves the concept of bands of allowed energy levels containing vacant levels into which electrons may be excited. Professor Raynor does an excellent job of leading the reader into this rather difficult subject. His discussion on nickel would have been more in keeping with the general level of the rest of the article if such terms as para- and ferromagnetic, Bohr magneton, and saturation magnetic moment had been defined.

The second paper by J. A. Catterall covers some experimental aspects of the electron theory of metals. Dr. Catterall outlines experiments used to investigate the energy band. His treatment is good with the exception of discussions involving the "Fermi surface" and Brillouin zone." Neither of these terms is defined in this or the preceding article by Raynor.

The third paper by A. G. Quarrell concerns dislocations in metals. The author presents a clear non-mathematical treatment of the subject. Many of the classical sketches of various configurations of dislocations are given. In his discussion on dislocation jogs, Professor Quarrell considers two dislocations AD and XY on two different slip planes cutting through each other to produce a jog. A sketch would have been of use in making the explanation more clear.

The fourth and last paper by J. Nutting is entitled "Seeing Dislocations." A very substantial effort is being expended in this field and has resulted in confirmation of the theory in general. Some ideas have, of course, been shown to be wrong, and other new ones have arisen. At the end of his article Dr. Nutting describes the results obtained from the direct observation of dislocations by transmission microscopy. As might be expected in such a rapidly advancing field, some of the reported findings may already now be considered as somewhat preliminary. For example, the expressed doubt as to the operation of Frank-Read dislocation sources in thin foils has been proven to be unjustified.

The author illustrates the text with figures and with plates. Unfortunately, the text and the plate numbers don't agree. The text refers to plates 1 through 18, whereas the plates are numbered 1*a*, 1*b*, 2*a*, 2*b*, etc. up through 12*b*. Plate 1*b* should be plate 2, plate 2*b* should be plate 4, etc. This is apt to cause some confusion to someone not reading the entire paper.

R. L. SMITH
The Franklin Institute Laboratories

BOOK NOTES

PROPERTY MEASUREMENTS AT HIGH TEMPERATURES, by W. D. Kingery. 416 pages, diagrams, 7 × 10. New York, John Wiley & Sons, Inc., 1959. Price, $16.50.

The subtitle—Factors Affecting and Methods of Measuring Material Properties at Temperatures above 1400°C (2550°F)— indicates the scope of this work. Although an introduction to the study, the book is intended for use on the graduate level or above. Each chapter opens with a section defining terms and reviewing fundamental concepts which were presumably learned in prior courses. Fourteen chapters cover measurement in general (with descriptions of instruments and techniques), laboratory refractories and furnaces, heat conduction and radiation, density and thermal expansion, and properties of solids at high temperatures. References at the end of each chapter are quite complete, and are divided into subjects related to the material in the chapter. The final chapter, Surface Energy, deals with high temperature surface phenomena, covering measurements of liquid surface tension and solid surface tension. This well written and well organized work should prove a most useful reference for the increasing number of students and scientists interested in research on high temperature phenomena. It is the first book to compare the different techniques of high temperature measurement.

FUELS AND LUBRICANTS, by Milosh Popovich and Carl Hering. 312 pages, illustrations, 5¾ × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $8.50.

In contrast to most books on fuels and lubricants, this one emphasizes performance characteristics and the significance of standard tests, rather than production. Prepared as a college text, the book is also useful to men in the industry. Thirteen chapters cover solid fuels, gaseous fuels, gasoline, lubricating oils, solid lubricants, rocket propellants and nuclear fuels, as well as such general topics as calorimetry, organic chemistry, petroleum refining, and residual fuels A useful syllabus of tests (both ASTM standards and Federal Test Method Standard 791) is included, with a summary of each test method.

ANALYSIS OF PIPE STRUCTURES FOR FLEXIBILITY, by John Gascoyne. 181 pages, diagrams, 7 × 9¾ in. New York, John Wiley & Sons, Inc., 1959. Price, $7.50.

This book, intended as an aid to piping designers, presents a survey of the more elementary theoretical methods for analyzing stresses due to thermal expansion in relatively simple piping systems, and practical means for minimizing such stresses. The main analytical tool is the method of elastic centers, of which a clear exposition is given, as well as a number of completely worked-out examples. Descriptions, photographs and drawings of a variety of pipe supports, and flexible and expansion joints are shown as a help to translate analytical results into workable systems. Also included are relevant excerpts from both British and American Codes and Standards, and tables of thermal properties of piping materials.

ELECTRONIC COMPONENTS HANDBOOK, edited by Keith Henney, Craig Walsh and Harry Mileaf. Vol. 3, 180 pages, diagrams, 8½ × 11 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $10.00.

Designers of military and commerical electronic equipment will find in this third and final handbook in the series on electronic components, more of the factual information needed for wider use of military electronic equipment. Throughout this text, emphasis has been placed on component types where military specifications exist, but other types are included as well. Data are also provided on transformers, amplifiers, saturable reactors and connectors.

NEW INSTRUMENTS AND METHODS OF ENGINEERING GEOLOGY, by N. V. Glazov and A. N. Glazov. Translated from the Russian by J. Paul Fitzsimmons. 91 pages, diagrams, 6 × 9 in. New York, Consultants Bureau, Inc., 1959. Price, $3.25.

This new treatise translated from the Russian, discusses not only methods employing radioactive elements, but cites also numerous other methods based on various physical principles. The authors have selected among the instruments and methods that use the properties of radioactive isotopes, the simplest in operation and the most promising for effectiveness of investigation. The reported data by no means cover completely the problems that are encountered in practice, but to some extent will facilitate further improvement in performing engineering geological and hydrogeological studies leading to wider application of more advanced methods and up-to-date equipment.

HANDBOOK OF SUPERSONIC AERODYNAMICS, edited by Stuart L. Penn. THREE-DIMENSIONAL AIRFOILS, Section 7 of the *Handbook*. 79 pages, diagrams, 8 × 10½ in. Washington, U. S. Government Printing Office, 1959. Price, $1.50.

Section 7, Volume 3, of the *Handbook of Supersonic Aerodynamics* deals with three-dimensional airfoils. The material is presented in a form which can be used for design calculations for a limited class of uncambered wings with zero angle of attack for which thickness ratio and position of maximum thickness are held at a constant value. No reference is made to the bodies to which the wings are attached. Supersonic source distributions are used to compute theoretical airfoil characteristics for lift, pitching moment, and pressure drag. The results are presented in graphical form, with a minimum of discussion on theoretical analysis. Sample calculations are given to show how to use the graphs.

STEAM POWER PLANTS, by Charles Donald Swift. 491 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $11.50.

This is a practical guide to all phases of steam power plants—from design, through construction, operation, maintenance and testing. The author's thirty years of experience with large utility electric generating stations and with many institutional and industrial power plants lends authority to his presentation. There are chapters devoted to specific topics such as boiler auxiliaries, water supply and treatment systems, condensers, electrical systems, automatic control, economy tests, etc. There are no lists of references included.

ELECTRIC ENERGY CONVERSION, by Y. H. Ku. 522 pages, diagrams, 6 × 9 in. New York, The Ronald Press Company, 1959. Price, $10.00.

This new text treats analysis of electrical machinery and power networks with emphasis on fundamental physical principles and analytical techniques. Written primarily as an introductory text for juniors and seniors, the book can be profitably used on the graduate level. Prerequisites are vector analysis, transform calculus, tensor analysis and a working knowledge of instantaneous components.

The text, covering both a-c. and d-c. machines, relates all electromechanical machines through power and torque. Equivalent circuits are stressed, to provide a tool for the analytical development of both steady-state and transient behavior. Several somewhat neglected machines are covered, such as a-c. servomotors, capacitor motors, drag-cup tachometers and universal motors. Separate chapters are also devoted to the analyses of self-synchronous machines and systems, and to the Metadynes. The text should add strength to the revival of interest in the improvement of electrical machinery.

PETROLEUM REFINERY MANUAL, by Henry Martyn Noel. 182 pages, diagrams, illustrations, 7 × 10 in. New York, Reinhold Publishing Corp., 1959. Price, $7.95.

This volume presents a comprehensive picture of refinery design and construction, including descriptions of all standard techniques of petroleum processing. It provides complete information on planning and constructing new refineries including preliminary considerations of engineering, shop fabrication and field construction. An attempt is made,

as a special feature of this book, to specify the number of man hours required for each phase of new refinery construction. More emphasis is placed on newer processes; however, former methods are outlined for comparison purposes. This complete manual bases its information on both large and small refineries accompanied by figures covering capital investment, operating manpower and costs, as well as descriptions and diagrams of all modern refining processes.

AN INTRODUCTION TO PLASTICITY, by William Prager. 148 pages, diagrams, 6 × 9 in. Reading, Mass., Addison-Wesley Publishing Company, Inc., 1959. Price, $9.50.

Based on a series of lectures delivered by the author at the Federal Polytechnic Institute in Zurich, Switzerland, this work has been designed to supplement traditional texts on strength of materials and theory of structures primarily concerned with elastic behavior. The scope of the book is restricted to selected topics from the theory of perfectly plastic solids and brings the reader up to the frontiers of current research without presupposing prior knowledge of the subject. Much of the material presented is of fairly recent origin, not found in standard reference works.

KINETICS OF HIGH-TEMPERATURE PROCESSES, edited by W. D. Kingery. 326 pages, illustrations, 8 × 10½ in. New York, John Wiley & Sons, Inc.; Cambridge, The Technology Press; London, Chapman & Hall, Ltd.; 1959. Price, $13.50.

This volume is comprised of 35 papers presented at the June 1958 Endicott House Conference at M.I.T., dealing with the kinetics of condensed-phase processes at elevated temperatures. Participants from five countries attacked the subject from several vantage points—physics, chemistry, metallurgy and ceramics. The papers are divided into seven sections: Imperfections and Diffusion in Nonmetals (10); Diffusion in Liquids and Liquid–Solid Reactions (4); Nucleation and Growth (5); Sintering and Vitrification (5); Phase Transformations (4); Solid–Solid Reactions (3); and Solid–Gas Reactions (4). Each section is prefaced by a short introduction and is followed by a discussion; each paper has a list of references, some as late as 1958. This is an excellent progress report on the status of the art and its usefulness lies not only in the information it presents, but in the fact that it points out areas in which further research may prove fruitful.

SOURCE BOOK OF INDUSTRIAL SOLVENTS, by Ibert Mellan. 276 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1959. Price, $10.00.

This is the third volume in a series presenting a compilation of data on industrial solvents aimed largely at quantity users. It emphasizes for each solvent manufacturing methods, physical properties, azeotropic mixtures, and some uses with a somewhat arbitrary inclusion of additional miscellaneous data. The data are not well referenced (121 references pertaining to about 50 solvents) so the book may be primarily considered a handy quick reference source rather than a thorough treatment.

FREQUENCY RESPONSE FOR PROCESS CONTROL, edited by William I. Caldwell, Geraldine A. Coon and Leslie M. Zoss. 395 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $11.50.

Fundamental methods of frequency response and their application to the analysis, testing and design of process control systems, are clearly presented in this newly published text. It thoroughly explains the analysis of complete systems by means of highly effective techniques giving typical solutions to many practical problems. Closed-loop responses are explained and disturbances discussed with regard to their magnitude and location in control loop. These fundamentals are applied to actual process control problems in Part II. Methods for calculating system time constants, along with useful material on the presentation of data and the dynamics of temperature measurement, are discussed. The final chapter presents a technique for converting from step to frequency response using a desk calculator.

PHENOLIC RESINS, by David F. Gould. 213 pages, diagrams, 5 × 7¼ in. New York, Reinhold Publishing Corp., 1959. Price, $5.75.

This small handbook gives a concise presentation of the raw materials, production methods and applications of the phenolic resins. Emphasis is placed on practical features as well as commercial and economic considerations by describing each major field in separate chapters. Many patent disclosures serve as a basis for the development of new uses for the phenolics, and also information for combining with other resins. Here is presented for engineers, designers, architects and management, a complete treatise on the present and potential uses of phenolic resins.

PUBLICATIONS RECEIVED

MATHEMATICS OF FINANCE, by Edwin D. Mouzon, Jr. and Paul K. Rees. 464 pages, 6 × 9 in. Boston, Allyn and Bacon, Inc., 1959. Price, $7.00.

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Solid-State Computer.—Pioneering a revolutionary concept in electronic data processing systems, Burroughs Corporation has announced a high speed, solid-state computer which is called the Burroughs B251 Visible Record Computer, specifically designed for the banking industry to handle the largest data processing problem in the world. Although the system will have other applications, the company's marketing plans call for concentrating first on the bank market.

Named the Visible Record Computer because it selects and writes directly on individual account records, the computer reads information directly from checks, deposit slips and other documents of varying thicknesses and sizes, processes the information and automatically computes, selects and posts directly to the correct customer's account record.

The system utilizes Magnetic Ink Character Recognition (MICR), the term for numbers that can be read by electronic machines as well as by people. MICR was announced last winter by the American Bankers Association as the "common language" for that industry.

A marriage of high speed electronic computation, advanced magnetic techniques and automatic handling of accounting forms, the system will automatically perform all of the work in a bank's bookkeeping and statistical analysis departments, including check handling, proving, bookkeeping and preparation of customers' statements and bank records.

Flexibility of the system will permit banks to use it for checking and savings accounts, installment loans, mortgages and other accounting operations. It will also provide accounting and statistical reports vital to bank management.

Four units comprise the system. The first is the world's fastest sorter, a solid-state device announced by Burroughs last March. This machine sorts and reads checks, deposit slips and other bank media through the means of the magnetic ink characters printed on these items, including the account number, amount and other required information.

The second unit is the computer, a completely transistorized machine with magnetic core memory. The computer processes the information read by the sorter from the magnetic characters on the documents and sends it back to the third unit, the record processor. Here items are automatically posted on ledger and statement forms, proved and the customer's current balance computed and printed.

The Visible Record Computer is controlled from a console, fourth unit of the system. It consists of program selector knobs, special controls, a panel of lights that displays status of the computer, and a keyboard which permits entry of unusual items if desired.

Instructions can be given the system to reject checks on which stop payment orders have been placed and refuse payment of checks if an account is overdrawn. It also can accomplish balance transfers, compute and post service charges and account profitability information and accumulate trial balances all at the same time.

As a by-product of processing operations, the computer system will provide accounting and statistical data needed for management decisions, including reports on inactive, overdrawn and closed accounts as well as significant balance changes.

Individual account information printed in standard type on the face of each ledger is also stored electronically in two narrow magnetic stripes on the back of the form. This enables the machine to read the ledger as well as carry out special instructions also stored in the stripes.

The pair of stripes, performing the same function as magnetic tapes used by other computers for input and storage of data, have a capacity of seven "words" of information. (A computer "word" consists of 12 digits plus sign.) They will contain such information as account number, balance, account activity, check and deposit counts and numerous computer commands.

Computer programs are stored on long-lasting Mylar plastic tapes, which are easily installed on tape readers in the control unit. The system may use up to 12 tape readers, permitting more than 2500 programming instructions at any one time. A library of standard programs covering a range of applications will be provided by Burroughs to banks using the new computer system.

Storage of programs and individual account information on Mylar tapes and magnetic ledger stripes leaves core memory of the computer free for intermediate storage of data, totals and constants needed for arithmetic functions.

The system will utilize advanced new memory core packets, each with a capacity for storing 10 computer "words." This will permit a bank to tailor capacity of the computer to its individual needs, saving the expense of buying core storage that will not be used. As a bank's requirements grow, the core memory capacity can be increased simply by inserting additional memory packets into the computer.

In addition to the Visible Record Computer's low initial cost compared to other automatic data processing systems, installation costs are held to a minimum. Specially-built rooms, air conditioning and humidity controls required for most computers are unnecessary.

The system is simple to operate and requires only one person in attendance. It is capable of performing 4000 arithmetical functions a minute.

A wide-line printer with 160 print positions is used to up-date ledgers and prepare other visual records. It prints at the rate of 200 lines per minute. Printing format is controlled from a plugboard in the console.

Translucent Polycrystalline Ceramic.—A new ceramic material described as "history-making" in its unique combination of properties has been announced by Dr. Guy Suits, vice president and director of research for the General Electric Company.

"For the first time in history," said Suits, "a polycrystalline ceramic that readily transmits light has been made from powders. In addition to transmitting light, the new material possesses the extremely high strength characteristic of alumina ceramics, can withstand much higher temperatures than most ceramics now in use, and can be pressed into any shape desired. No other material combines translucence with this degree of strength, heat resistance, and ease of fabrication."

"Lucalox," as General Electric calls the new material, is made from powdered aluminum oxide. It is closely related to sapphire and ruby gem

stones, which are single-crystal aluminum oxide. But GE's polycrystalline form of the same compound is superior to these gems in its ability to withstand high temperatures without deforming, according to Suits. Lucalox has the composition of a ceramic, the structure of a metal, and the light-transmitting ability approaching that of glass, he added.

The unique characteristics of Lucalox result from the fact that the microscopically small pores, or "bubbles," that are normally found in ceramic materials have been entirely removed, a feat that had been previously considered impossible.

Because of the elimination of the pores, it is possible to read through a sheet of Lucalox when it is laid flat upon a piece of paper. At greater distances it appears translucent, resembling frosted glass. At least ninety per cent of the light in the visible spectrum is transmitted through the new ceramic.

The basic material of Lucalox is fine-grain, high-purity aluminum oxide, or "alumina." The powder is pressed at room temperature, then fired at temperatures that are higher than usual for ceramics.

"The development of Lucalox," according to Dr. J. Herbert Hollomon, of GE's Metallurgy and Ceramics Research Department, "should greatly enlarge the scientific, military, and industrial uses of ceramics. As has been the case with many new materials, Lucalox should also extend the range of instruments and devices that are presently limited by the physical characteristics of available materials. One example would be high-intensity incandescent and discharge lamps, which are now limited by the heat-resistance of their transparent envelopes, in some cases. Fused quartz, which is often used for high-temperature lamps, performs satisfactorily up to 1800° F.; Lucalox is stable at temperatures close to 3600° F."

Another likely application for Lucalox is in the banks of infrared lamps that are used to test the heat-resistance of missile nose-cones and other space-vehicle equipment. It may also be used as an electrical insulator and as a material for gem bearings in delicate equipment, according to Hollomon.

Lucalox has a metal-like structure, in that its crystals are bonded directly to one another, with neither pores nor a glass matrix between them. Information gained from the study of this material may consequently be applied to powder metallurgy.

Perching Birds Damage Missile Sleds.—The famed sled track at Holloman AFB, New Mexico, has been "wired for sound" in an attempt to frighten away the many birds that perch on the glistening rails of the 35,000-ft. missile track.

The Air Force Missile Development Center reports that unbelievable damage has occurred to sleds that have hit birds while traveling at speeds of more than 3000 mph, down the 7-mile track at this Air Research and Development Command base.

The birds cannot hear the sleds approaching at faster-than-sound speeds so recordings of machine-gun blasts, sounds of thunder, and hawk cries are being amplified by speakers up and down the track. The test supervisor, R. C. Rethmel, reported, "We have just started this method of trying to scare the birds away and the results have not yet been proven."

A 212-lb. monorail sled traveling 3000 miles per hour recently ended its hypersonic run with a 10-inch jagged hole ripped completely through the ¼-in. steel of its nose cone. The im-

pact of a bird into a heavy dual-railed sled during an 850-mph. run tore loose ½-inch sheeting at the bolted seams.

Additional incidents of this kind included a four-inch rupture in the wedge-like steel prow of a sled speeding at 2850 mph. A 375-lb. sled was also twisted while traveling at 1500 mph.

According to Lt. Col. Donald H. Vlcek, Chief, AFMDC's Test Track Division and the man mainly responsible for the track and its super-modern equipment, "high velocity impact" causes this particular phenomenon in the same way that lighter materials may be driven through solid tree trunks by a hurricane's force, or grain-sized meteorites moving through space at near-ultimate velocities could tear apart the solid steel of a missile or a space craft.

Meanwhile, Holloman's sleds are still moving, though, as Colonel Vlcek remarked: "We'd have one less technical headache if somebody came up with the perfect answer about what to do with the birds." In the meantime, the technicians at Holloman are trying to aware the birds from their rail-perching pastime with all the weird sounds they can find.

Low-Cost Pulsed Neutron Generator.—A new pulsed neutron generator, low in cost and within the budget reach of all institutions engaged in research with sub-critical reactors, has been announced by Atomic Laboratories, Berkeley, Calif.

The new unit, called the Omnitron, is simpler in construction than other pulsed neutron generators, thus making it easier to operate and maintain. In addition, it is compact and portable, inexpensive to operate, and requires no special installation or power facilities.

Three functions can be performed by the Omnitron while operating contiguous to such objects as sub-critical assemblies and training reactors:

1. Provides a graphic demonstration of fusion reactions and neutron counting techniques.
2. Makes radio-isotopes by neutron activation.
3. Permits the study of the transient behavior of neutrons in various multiplying and non-multiplying media.

Although most pulsed neutron generators require an expensive vacuum system, the Omnitron operates satisfactorily with an inexpensive mechanical high vacuum system.

If a diffusion pump system is desired, several Omnitron models come with the best high vacuum units available. Yields in excess of 100 million neutrons per second are possible with a mercury diffusion-mechanical vacuum pump combination.

Basic components of the Omnitron are: high voltage pulse transformer; deuterium neutron target and acceleration tube; and remote control electronic rack.

Price of the Omnitron, without vacuum system, is $4950. With mechanical pump, cold trap and manifold in open frame, the unit sells for $7700. Higher priced models containing diffusion pump units and additional equipment are also available.

Catadioptric Light Screen Sensitive to Hypervelocity Projectiles.—A new light screen sensitive to hypervelocity projectiles as small as .22 caliber that uses the catadioptric technique of reflection and refraction has been developed by Avco Research and Advanced Development Division of Wilmington, Massachusetts.

The device sets up a screen of light consisting of a collimated light beam

which is reflected many times between parallel mirrors before passing into a photoelectric detector. The effective width of the beam interrupted by a projectile is three-quarters of an inch in any part of a 6 × 6-in. field. This gives a maximum phototube output signal over a wide range of projectile sizes. A current change is caused in the detector when a hypervelocity projectile breaks the light screen at any point. The current change is amplified and is used to trigger a shadowgraph system.

Shadowgraph systems used on the Avco ballistic ranges produce photographic records of missile models in flight. The shadowgraph records are used to measure missile attitudes and aerodynamic flow. Velocity and drag can also be studied by measuring the time of flight between two light screens with standard chronometer techniques.

The photoelectric detector consists of a photomultiplier tube which provides a low-output impedance from the light screen and permits use of a low-gain amplifier with associated reduced microphonic characteristics. Baffles are used on the lower part of the screen to minimize stray light reflection and other interference to the collimated light light beam. The light screen being produced by Avco measures 9½ in. high by 10 in. wide by 6½ in. deep. It has a uniformly sensitive screen area of 6 × 6 in. And, the height of the opening can be enlarged by substituting longer support rods.

Other hypervelocity instrumentation in the Avco shadowgraph system are a high-intensity light source and a Kerr Cell photographic shutter. The system allows exposure times in the range of 10^{-8} and 10^{-7} second.

Amplifier Improves Vibration Data Analysis.—A new amplifier, designed for application involving the amplification of signal from sources having high internal impedances, has been released by Columbia Research Laboratories, Woodlyn, Pa.

The unit, the Model 6000, provides the impedance matching and precise gain control necessary for the measurement of shock and vibration over the broad dynamic ranges required by the military.

The instrument provides faithful reproduction of accelerometer signals over a broad frequency band from 1 cps to 1 mc at input levels up to 20 volts and features variable gain from .3 to 100. Input impedance is 1600 megohms and output impedance is 150 ohms. With an accelerometer connected to the input and the gain set at 100 the input noise level is less than 40 microvolts.

Special front panel controls allow calibration of accelerometer signal so that sensitivities may be normalized in order to simplify oscilloscope readings.

An input shut capacitor switch provides a means of extending the effective input overload voltages by as much as one hundred times for high level shock measurements, and with a fixed gain setting it provides a practically flat accelerometer sensitivity with temperatures to 500 F.

Material Developed for Space Shields.—A totally new material capable of protecting man, animals or delicate instruments in the awesome flight through space and re-entry into the earth's atmosphere has been developed by the General Electric Company's Missile and Space Vehicle Department, Philadelphia. The material was developed as a heat shield for nose cones sent into space with invaluable cargoes to protect them against the tortures of supersonic

speeds, searing heat and rigorous mechanical forces encountered in re-entry flight. Space shields made from the new material can be produced for a fraction of the cost of the protective coatings currently used on test flights into space, General Electric scientists said.

The material, an organic plastic substance, has undergone many tests in preparation for space flights expected to be conducted within the next few months. Space shields of the new material can be produced in relatively short time, compared to the weeks-long production schedule for current nose cone protectors. Production simplifications and cost saving will prove a decided advantage to the U. S. Government and to the taxpayer.

The two fabrication systems now in general use are more complicated. In one, the protective material is molded in high pressure presses. In the other, miles of special impregnated tape is wound around nose cones as a protective outer coating. Laminated and high pressure molded phenolic materials now used as nose cone thermal shields are difficult and costly to fabricate.

The new material differs radically from these previously known heat protective materials in that relatively inexpensive, easy to procure equipment can be used for its manufacture. The basic materials are blended and heated in temperature controlled ovens, mixed to the proper consistency, subjected to an air removal process, and poured into molds. Material molded in the new simplified process developed by General Electric could be designed for the *specific* job to be done, since it can be made elastic, flexible, or hard, without affecting its excellent heat protective qualities.

The space shield material has been successfully tested in high temperature facilities. In one such test, supersonic flame of more than 5000 F. is blasted at the material. In another, the material is subjected to temperatures of more than twice the surface temperature of the sun—as great as man has ever created on this Earth for sustained periods.

Electronic Thermometer.—A new type of electronic thermometer which will measure temperature immediately it is placed in contact with an object has been developed by Wayne Kerr Laboratories of Tolworth, Surrey, England. Results achieved so far suggest an important advance in clinical medicine—the virtually instantaneous measurements of the temperature of the human body within a second or so of its initial application to the skin's surface.

The device depends on the use of a temperature-sensitive metallic oxide lead which is connected to a sensitive direct current bridge measuring circuit. The complete unit weighs 4 lb. and is $8\frac{1}{2} \times 6 \times 4\frac{1}{2}$ in. One of its advantages is that the temperature probe can be operated at a distance of up to 1000 ft. from the recording device.

The thermometer, at present in prototype form, can be used also, it is claimed, to measure the heat lost from particular windows of buildings, doors, ceilings or office heating systems as well as during the different stages of industrial processes.

Penetrating the Earth's Crust.—The hot, viscous interior of the Earth can be reached and sampled within four years, according to a report of a committee of the National Academy of Sciences—National Research Council. Such a feat, declared the committee, will not only yield a vast store of scientific knowledge, but also may very well provide immediately

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practical information for the petroleum industry and oceanographers. The report also included the first detailed conception of how the operation —which calls for a 18,000-ft. borehole under several thousand feet of ocean water—may be carried out.

The project, which has come to be known as the Mohole, derives both its name and its goal from the discovery by Professor Andrija Mohorovičić, a Yugoslavian seismologist, of a transitional boundary between the Earth's outer crust and adjoining mantle at which seismic waves produced by earthquakes suddenly increase in velocity. This boundary, which geophysicists familiarly refer to as the "Moho," lies so close to the Earth's surface at certain points on the ocean floor that it seems to be within reach of present-day offshore drilling techniques, once the problems of drilling in the deep ocean can be solved.

This opportunity, long a topic of animated discussion among geophysicists, appeared extraordinarily attractive to a group of American geophysicists who had banded loosely together in an organization called the American Miscellaneous Society. Obtaining a sample of the Earth's mantle would not only provide invaluable clues to the origin and structure of the Earth, they recognized, but would also add most considerably to the meaning of great stores of geophysical data accumulated during the past several decades. In addition, coring techniques could be used during the drilling operation to bring up samples of the ocean's sedimentary floor that might well reveal the nature of marine organisms back through the ages to the origin of life itself.

The committee's optimism regarding the feasibility of the drilling operation is based in large part on the techniques that have been developed in drilling for oil off the Gulf and Pacific coasts. Although present-day methods appear to be immediately adaptable for the conduct of the first experimental holes, the committee notes in its report that drilling through approximately 30,000 ft. of ocean water, sedimentary floor and hard crust will probably require the adoption of several radical new techniques, together with several innovations that will have to be developed especially for the project and tested at the site. Among these are the use of turbo-drills and sound drills, the use of sea water as a drilling fluid, aluminum pipe, wire-line corers and ship-positioning devices.

Miniature Super-Cold Infrared Detector.—A new super-cold refrigerator the size of a flashlight battery promises significant improvements in the detection of enemy missiles and aircraft. The tiny cooler can be used to chill present airborne infrared target detectors down to −350 F. to make them ultra-sensitive, and therefore capable of spotting aircraft or missiles from far greater distances than is now possible.

Since all objects give off infrared rays similar to the invisible rays from household heat lamps, infrared detectors using the miniature refrigerator can also be employed in a wide variety of industrial and military applications.

The 8-oz. refrigerator, called the MinIRcooler, is the first commercial application of a new process for producing extremely low temperatures developed during the past two years by Dr. Howard O. McMahon, vice president, and William E. Gifford, of Arthur D. Little, Inc., industrial research and engineering consultants. The process and device were described by Dr. McMahon to scientists attending the Cryogenics Engineering Con-

ference at the University of California, Berkeley.

Dr. McMahon pointed out that cooling infrared detectors to extremely low temperatures increases their sensitivity and thus their range. It also makes the target detectors responsive to a wider range of infrared wave lengths, making it possible to detect smaller temperature differences between the "target" and its surroundings.

The ADL process and its ultimate application in a variety of small, efficient coolers is expected to unlock many significant new uses in addition to the infrared use, according to McMahon. He pointed specifically to cryotrons and solid-state masers, which operate in liquefied helium at −452 F. (4.2 K.). At present the equipment needed to maintain the helium at this temperature is bulky. When cryotrons are incorporated in miniaturized computers, supplementary refrigerating equipment will be of great importance. The Gifford-McMahon process will make it possible to recycle the evaporated helium in a closed system without using large cryostats. So, too, solid-state masers used for microwave amplification in radar and radio astronomy require a similar recooling system.

A complete airborne detection system based on the new cooler would weigh less than 20 lb. Further development work is expected to reduce this to less than half. The tiny cooler is unique in its simplicity, having no cold moving parts except a small plastic piston. In operation, pressurized helium gas expands in a $\frac{1}{4}$-in.-diameter cylinder 2 in. long. The expanding gas cools an infrared cell at the end of the cylinder.

The new process described by McMahon refrigerates to an exceedingly low temperature by passing compressed helium gas through a small "engine"—very much like an ordinary steam engine. The expanded helium, after it has been through the engine, and cooled, is used to cool the compressed helium, which is coming to the engine. Therefore lower and lower temperatures are achieved by this "bootstrap" process. The MinIR-cooler takes only 3 minutes to reach −315 F. (80 K.) by this process. The features of the process which make it uniquely advantageous are that it is simple, reliable, easily miniaturized and inexpensive.

New Wheatstone Bridge.—Leeds & Northrup Company, of Philadelphia, whose Anthony Pattern Wheatstone Bridge has served as a laboratory and production standard for accurate d-c. resistance measurements since at least 1903, is now offering a completely new and vastly improved 4232-B High Precision Guarded Wheatstone Bridge.

The old bridge's awkward and tiresome plugs and blocks have been replaced with enclosed rheostat dials and high-quality selector switches. The result is faster and easier operation by merely turning the dials and taking a direct reading from the windows above each dial.

Unlike its predecessor, the new bridge provides complete guarding which eliminates errors caused by adverse humidity conditions and resulting leakage currents. This guarding assures accuracies of ±0.01 per cent up to 1 megohm and ±0.02 per cent up to 100 megohms. The bridge has wide measurement range—from 0 to 11,111 megohms—or 100 times greater than the Anthony Pattern bridge's range.

Resistors may be tested with the new instrument at full battery potentials up to 100 volts, in accordance with the maximum values permitted in military specifications. Guard ter-

minals are also provided so that the bridge guard system can be completed with a guarded d-c. bridge power supply and a guarded detector. With each instrument L&N supplies a certificate based on its own high standards which are certified at intervals by the National Bureau of Standards.

The new bridge is already being used in some standardizing laboratories as the prime resistance measuring instrument for all d-c. resistance measurements. Resistor manufacturers are also using it to make accurate measurements on a semi-production basis.

Housed in a gray metal case, the bridge is designed for mounting in a 19-in. relay rack, while an optional set of brackets permits table-top mounting.

Ion Thrust Device.—An ion thrust device directed toward a prototype engine configuration has been in operation at Rocketdyne, a division of North American Aviation, Inc., for several months, producing "quantitative" measurements of ion thrust.

Dr. Robert H. Boden, program engineer on ion research activity at Rocketdyne, said the experimental ion device—believed to be the first such unit ever operated—has made numerous runs of varying durations since February, 1959.

He made the comments while presenting a paper on "Recent Developments and Designs of the Ion Rocket Engine" during the 10th Congress of the International Astronautical Federation at London, England.

"The tests have confirmed the analysis that ion engines will apparently be useful outer space power plants," Dr. Boden said.

He said a flyable ion engine, which would deliver less than 1 lb. of thrust but which could propel space vehicles on interplanetary voyages, could be ready within five years.

Rocketdyne's prototype ion unit operates in a vacuum tank which simulates conditions in outer space. Like the proposed flyable engine, the experimental unit produces a stream of charged particles similar to the stream of electrons in a television tube. This current provides the propulsive thrust.

First flyable ion engines will deliver a fraction of a pound of thrust, compared to the 360,000 lb. produced by the Atlas ICBM. Yet the small ion unit, which will operate continuously up to one year, will propel space vehicles thousands of miles an hour.

The ion engine—which will be approximately the size of an oil drum—will create ions and discharge them at tremendous velocities through a thrust chamber. Reaction to the stream of escaping ions will propel the space vehicles.

The ion engine is not planned to operate at sea level. But once propelled into space by a conventional chemical rocket engine, a one-pound-thrust engine could drive a 5-ton vehicle. In addition to interplanetary missions, the little engine, which could be used alone or in clusters, could also control huge space stations in orbit.

The engine will operate thusly:

A propellant—such as cesium which is being used in the current Rocketdyne research program—is vaporized and fed into an electrically charged chamber. There an electron is removed from each atom of vaporized propellant, leaving a positive ion.

The ions are then pulled out of the chamber by a high voltage electrostatic field, accelerated to velocities up to 300,000 mph., and higher if necessary, and then discharged in a stream through the thrust chamber, thus providing the propulsion.

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